UNIT-5

Vapour Power Cycle

Vapour Power Cycles

Introduction:

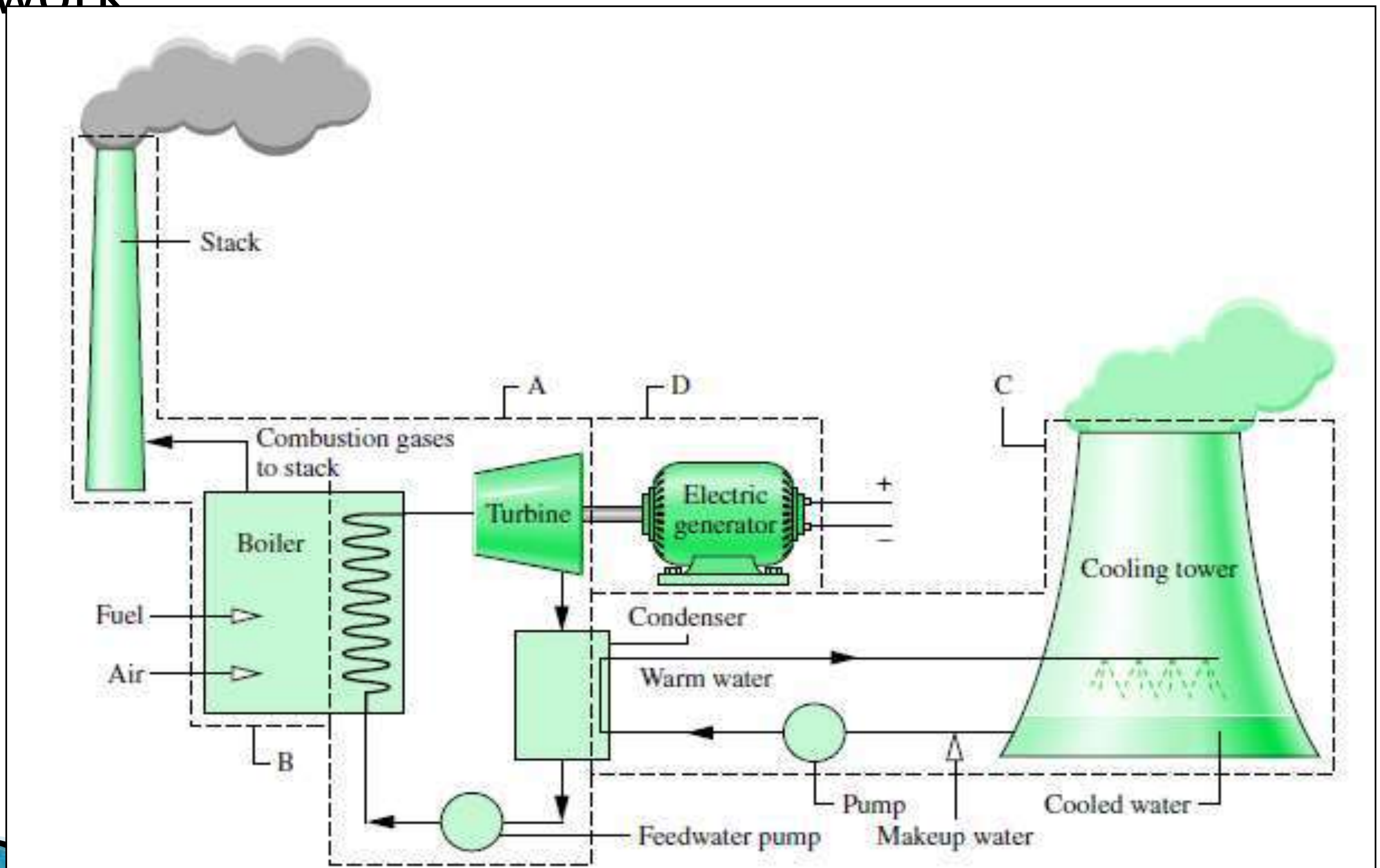
Vapour power cycles are external combustion systems in which the working fluid is alternatively vaporized and condensed.

Water/steam is easily available, is economical, chemically stable and physiologically harmless.

Hence it is the most commonly employed working fluid. Due to its use as working substance in vapour power cycle, this cycle is often referred as steam power cycle. The vapour is generated in a steam boiler which then enters the steam turbine, a condenser and a feed pump.

In a vapour power cycle, the main objectives are to convert the energy present in the fuels into mechanical energy and then to electrical energy.

The fuel is burnt, hot flue gases are used to produce steam in the steam generator. This steam so produced is expanded in a steam turbine to do work



A power cycle continuously converts heat energy into work, in which a working fluid performs a succession of processes. In the vapor power cycle, the working fluid, which is water, undergoes a change of phase into steam, which may be in the form wet, dry saturated or super heated. A vapor power plant differs from a gas power plant in that, it's working fluid may undergo a phase change during the working of the plant.

Like in any other power cycle, the working fluid (steam/water) in a steam power plant undergoes four basic operations in a cyclic manner.

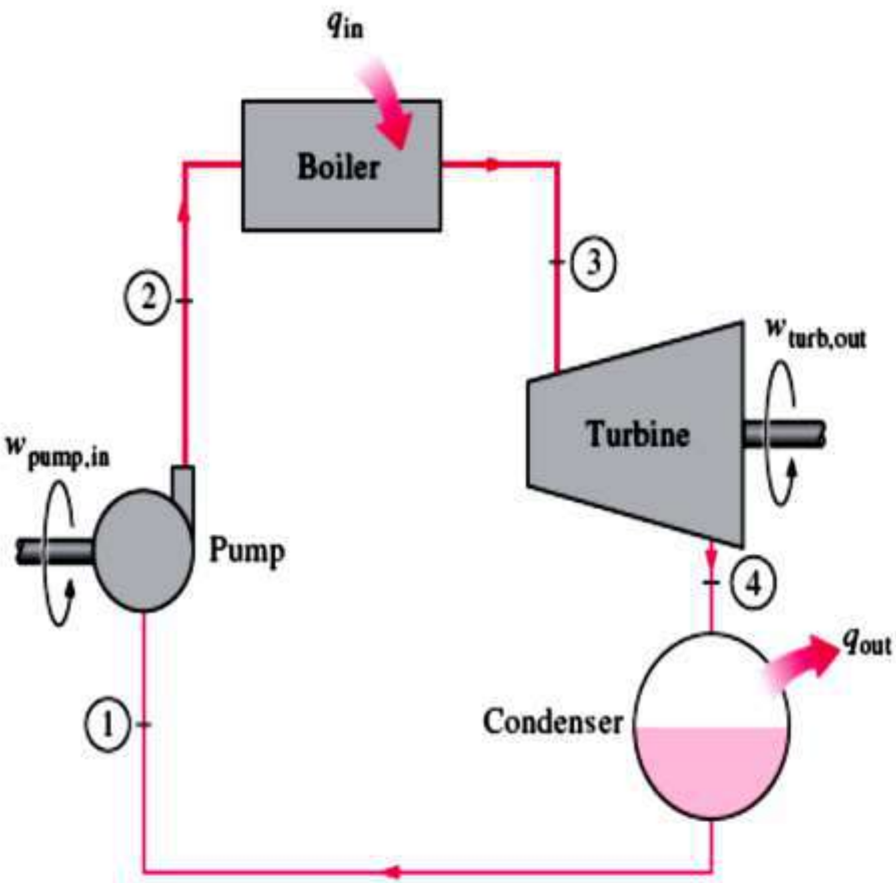
For each operation in a vapor power plant, we can think of a hypothetical or ideal process, which represents the basic intended operation. Since these operations are cyclic, the idealized processes representing these operations form an ideal cycle. That is known as vapor power cycle.

The Carnot vapor cycle:

A Carnot cycle with two isothermal and two isentropic processes can be thought of as a vapor power cycle.

However, in practice, it is almost impossible to design a vapor power plant, based on Carnot cycle.

Figure shows the schematic diagram of a simple steam power plant working on vapor power cycle along with p-v and T-s diagrams.

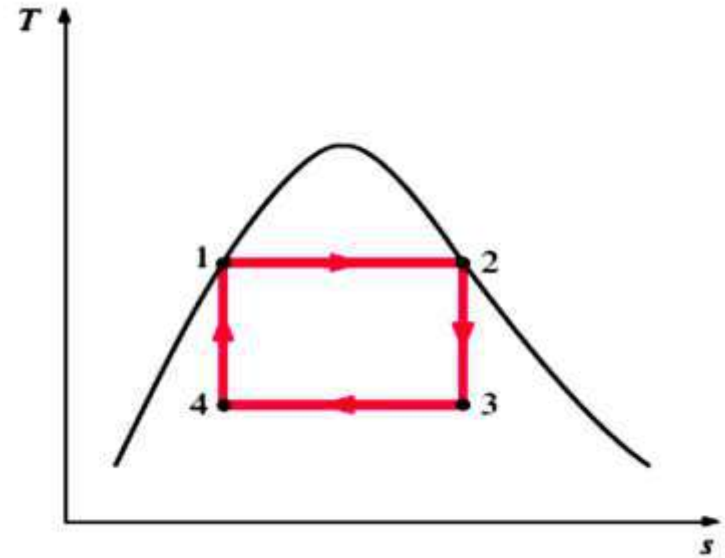


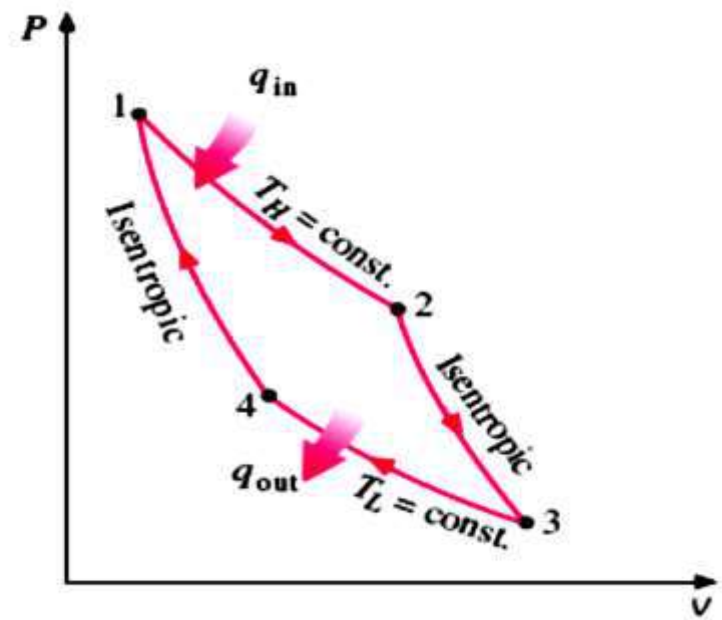
Heat is transferred to water in the boiler from an external source to raise steam.

The high pressure, high temperature steam leaving the boiler expands in the turbine to produce shaft work.

The steam leaving the turbine condenses into water in the condenser, rejecting heat and then water is pumped back to the boiler.

Consider 1kg of saturated steam at pressure p_1 and absolute temperature T_1, as represented by point 1. The cycle consists of four processes.





Process 1–2: The saturated water is isothermally converted into dry saturated steam in a boiler, at constant pressure p_1.

The dry state of steam is represented by point 2. The heat absorbed by saturated water during its conversion into dry steam is its latent heat of evaporation at pressure $p_1 = p_2$.

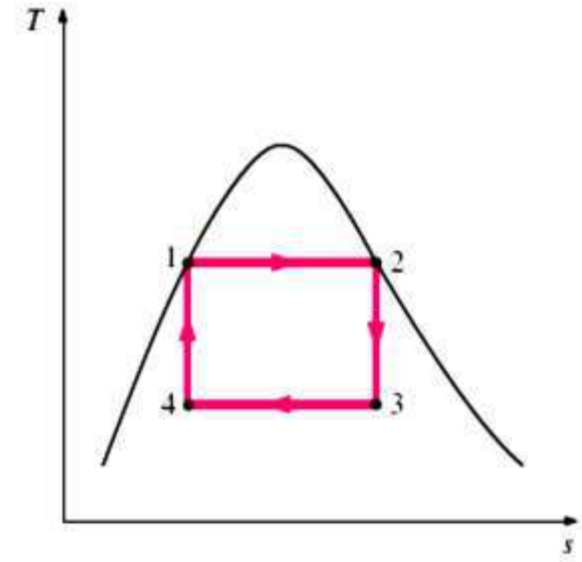
i.e.,

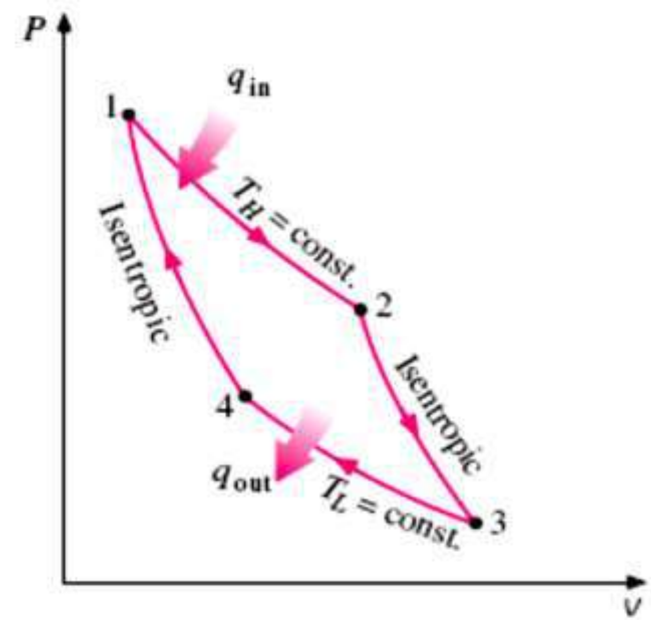
$$h_{fg1} = h_{fg2}$$

The area below the curve 1–2 in the T–s diagram represents the heat absorbed during the isothermal process.

Therefo

$$Heat\ absorbed = q_{1-2} = (s_2 - s_1)T_2 = (s_2 - s_1)T_1$$





Process 2–3:

The dry steam expands isentropically in steam engine or steam turbine.

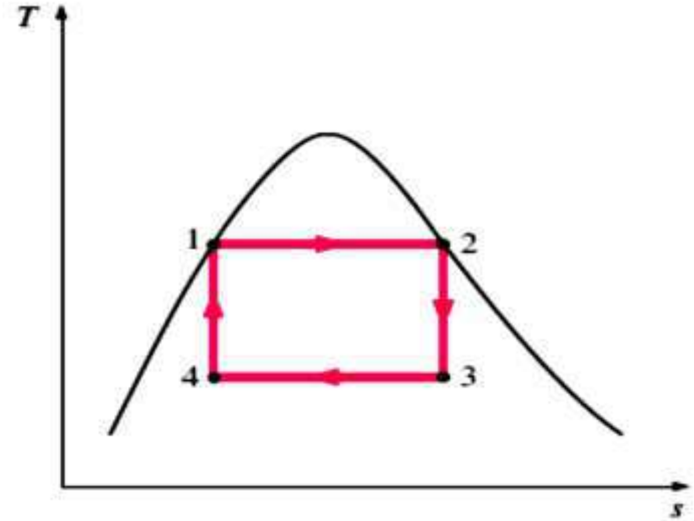
The pressure and temperature falls from p_2 to p_3 and T_2 to T_3 respectively.

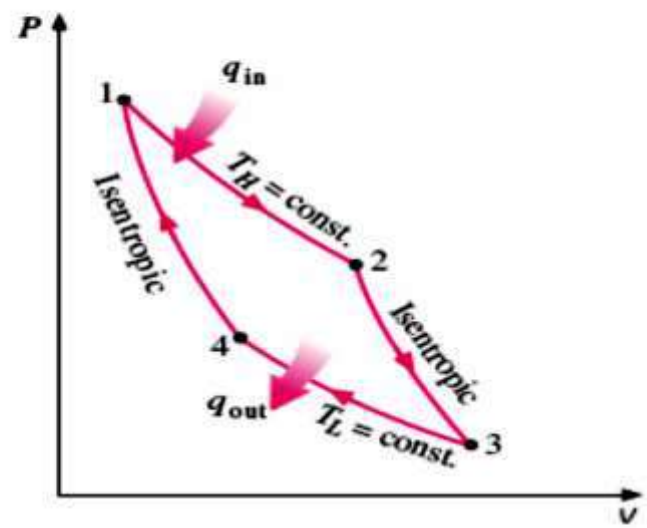
No heat is supplied or rejected during the process.

Process 3–4: The steam is now isothermally condensed in a condenser and heat is rejected at constant temperature T_3 and pressure p_3.

Here $p_3 = p_4$ and $T_3 = T_4$. The area below the curve 3–4 in the T–s diagram represents the heat

$$Heat\ rejected = q_{3-4} = (s_3 - s_4)T_3 = (s_2 - s_1)T_3$$





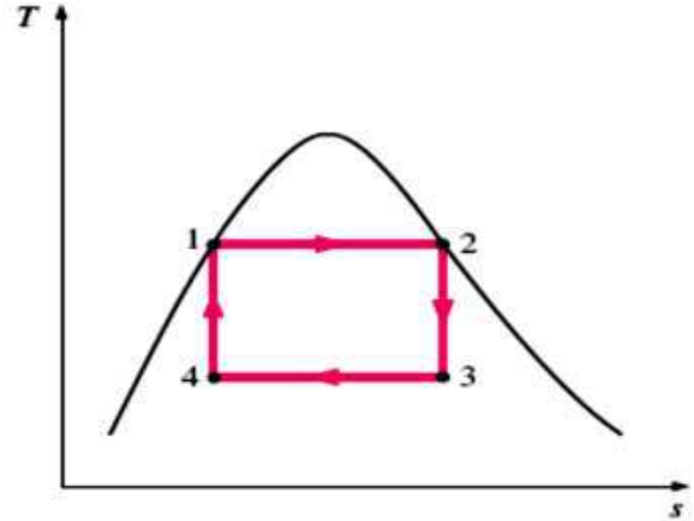
Process 4-1:

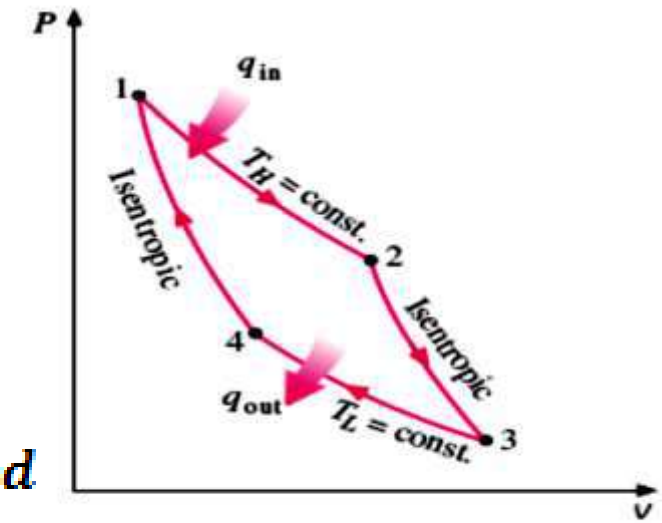
The wet steam at point 4 is finally compressed isentropically in a compressor, till it returns back to initial state 1.

The pressure and temperature rises from p_4 to p_1 and T_4 to T_1 respectively. Since no heat is absorbed or rejected during this process, therefore entropy remains constant.

Work done:

The work done during the cycle,





$$Workdone = Heat\,absorbed - Heat\,rejected$$

$$Workdone = (s_2 - s_1)T_1 - (s_2 - s_1)T_3 = (s_2 - s_1)(T_1 - T_3)$$

Efficiency of the cycle:

The efficiency of the Carnot cycle,

$$\eta = \frac{Workdone}{Heat\ absorbed} = \frac{(s_2 - s_1)(T_1 - T_3)}{(s_2 - s_1)T_1}$$

$$\eta = \frac{(T_1 - T_3)}{T_1} = 1 - \frac{T_3}{T_1}$$

Where, T_1 = Highest temperature corresponding to the boiler pressure $p_1 = p_2$.

T_2 = Lowest temperature corresponding to the condenser pressure $p_3 = p_4$.

Drawbacks of the Carnot cycle as reference cycle:

1. The isothermal processes 1-2 and 3-4 can be approached closely in actual boilers and condensers. Limiting the heat transfer processes to two-phase systems, however, severely limits the maximum temperature that can be used in the cycle (less than 374^{0}C for water).

   Limiting the maximum temperature in the cycle also limits the thermal efficiency.

   Any attempt to raise the maximum temperature in the cycle involves heat transfer to the working fluid in a single phase, which is not easy to accomplish isothermally.

2. During the isentropic expansion process 2-3 in the turbine the quality of the steam decreases.

Thus the turbine has to handle steam with low quality, that is, steam with high moisture content.

The impingement of liquid droplets on the turbine blades causes erosion and is a major source of wear.

Thus steam with qualities less than about 90% cannot be tolerated in the operation of power plants.

3. The isentropic compression process 4-1 involves the compression of a liquid-vapor mixture to a saturated liquid.

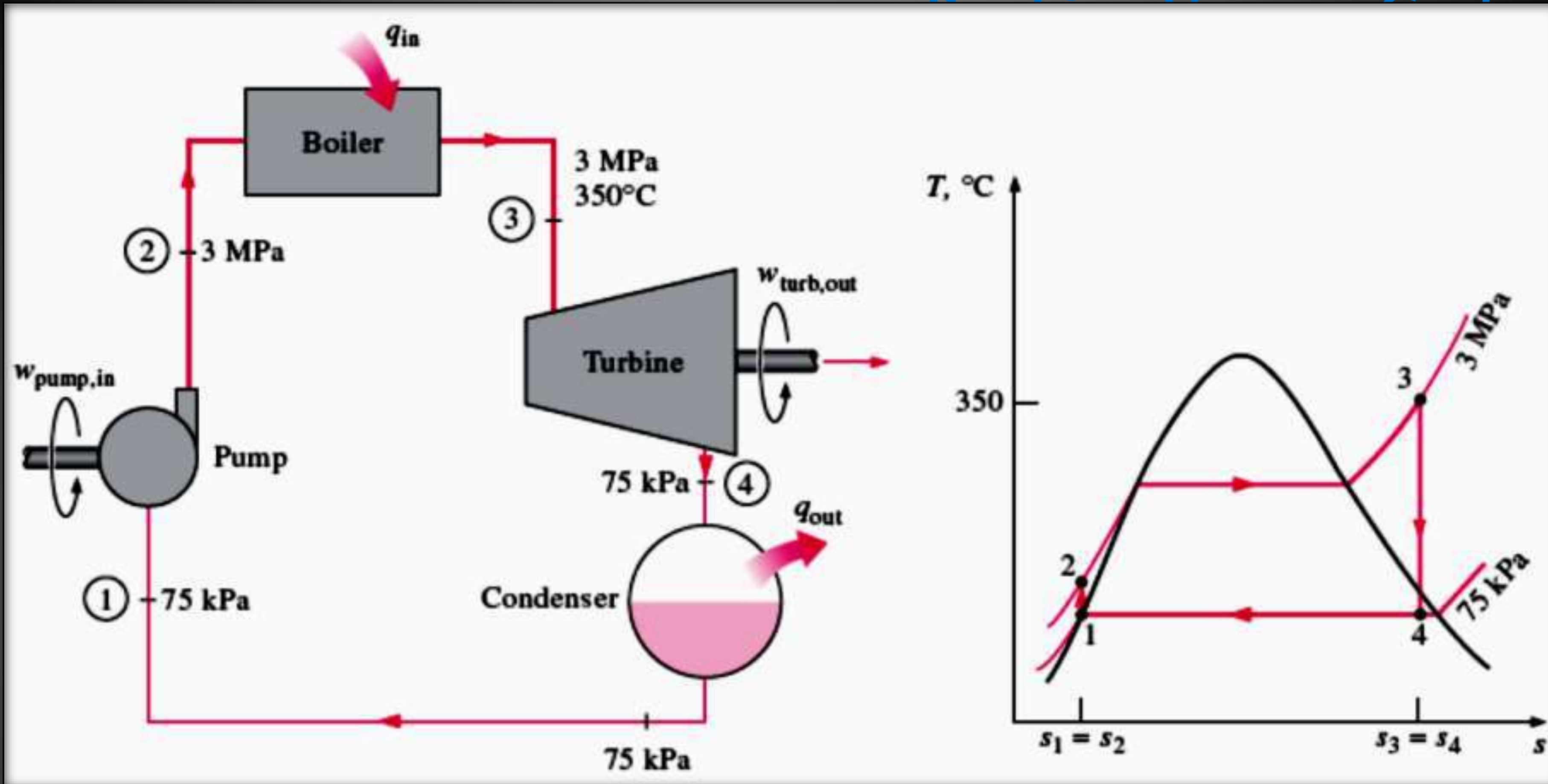
There are two difficulties associated with this process.

First, it is not easy to control the condensation process so precisely as to end up with the desired quality at state 4. Second, it is not practical to design a compressor that handles two phases.

Limitations of Carnot Vapour Cycle

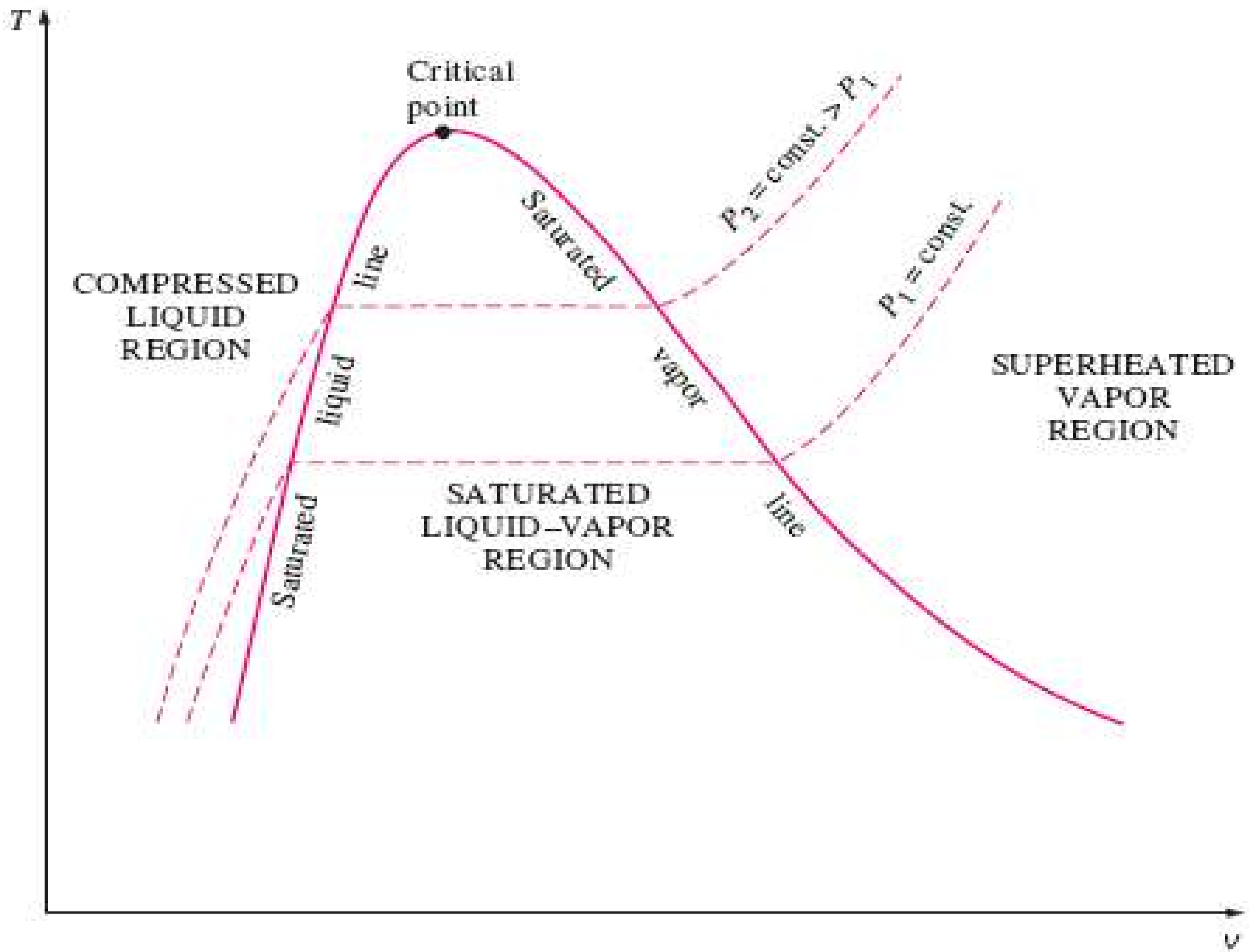
Theoretically the Carnot vapour cycle is most efficient; the following difficulties are associated with it during its operation.

1. Steam condensation is not allowed to proceed to completion. The condensation process has to be stopped at state point 3.
2. The working fluid at 3 is in both liquid and vapour state, it is difficult to compress two phase mixture isentropically.
3. The vapour has larger specific volume; hence to accommodate greater volumes, the size of the compressor becomes quite big.
4. For running a large sized compressor, more power is required; this results in poor plant efficiency.
5. The steam at exhaust from the turbine is of low quality i.e. high moisture content. The liquid water droplets cause pitting and erosion of the turbine blades.



q_{in}
Boiler
3 MPa
350°C
3
2
3 MPa
$w_{turb,out}$
Turbine
$w_{pump,in}$
Pump
75 kPa
4
q_{out}
1
75 kPa
Condenser
75 kPa
T, °C
350
3 MPa
3
2
1
4
75 kPa
$s_1 = s_2$
$s_3 = s_4$
s

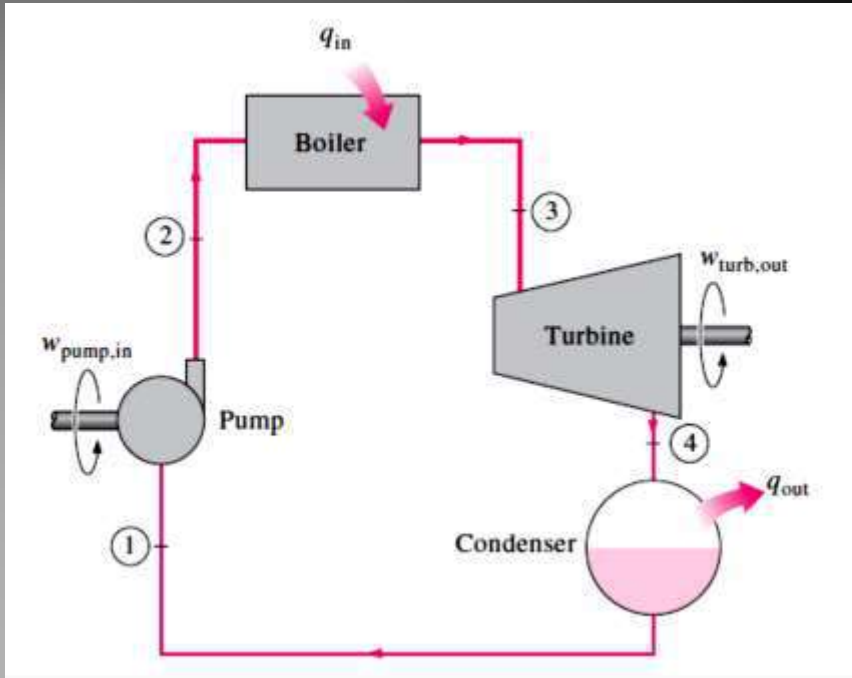
- # T–v diagram of a pure substance:

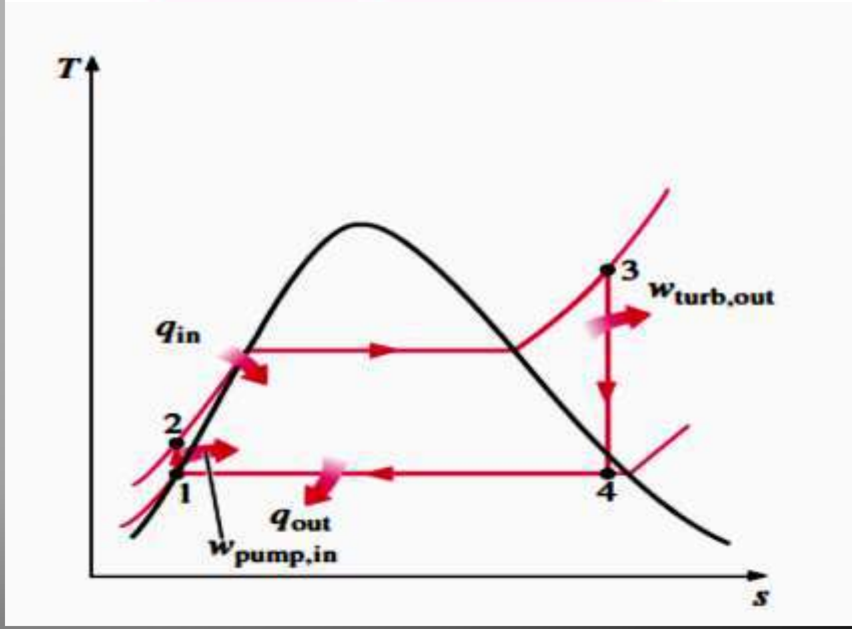


Process 1-2:

Isentropic compression in a pump: Water enters the pump at state 1 as saturated liquid and is compressed isentropically to the operating pressure of the boiler.

The water temperature increases somewhat during this isentropic compression process due to a slight decrease in the specific volume of water.

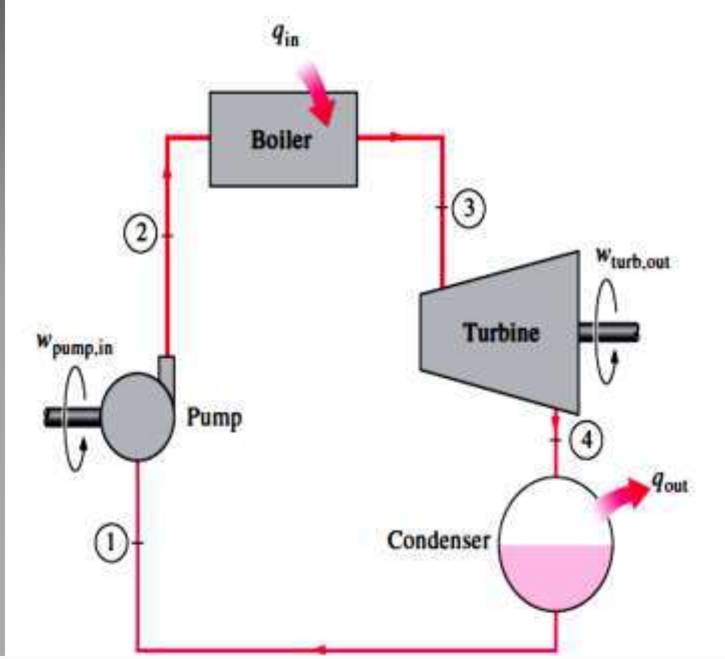


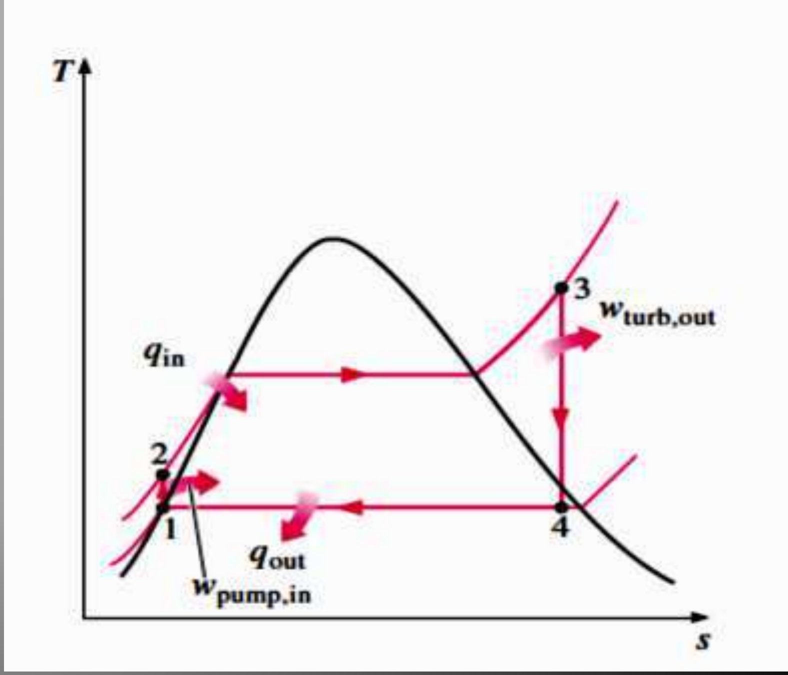


Process 2-3: Constant temperature heat addition in a boiler: Water enters the boiler as a compressed liquid at state 2 and leaves as a superheated vapor at state 3.

The boiler is basically a large heat exchanger where the heat originating from the combustion gases, nuclear reactors or other sources is transferred to the water essentially at constant pressure.

The boiler, together with the section where the steam is superheated, is often called as the steam generator.



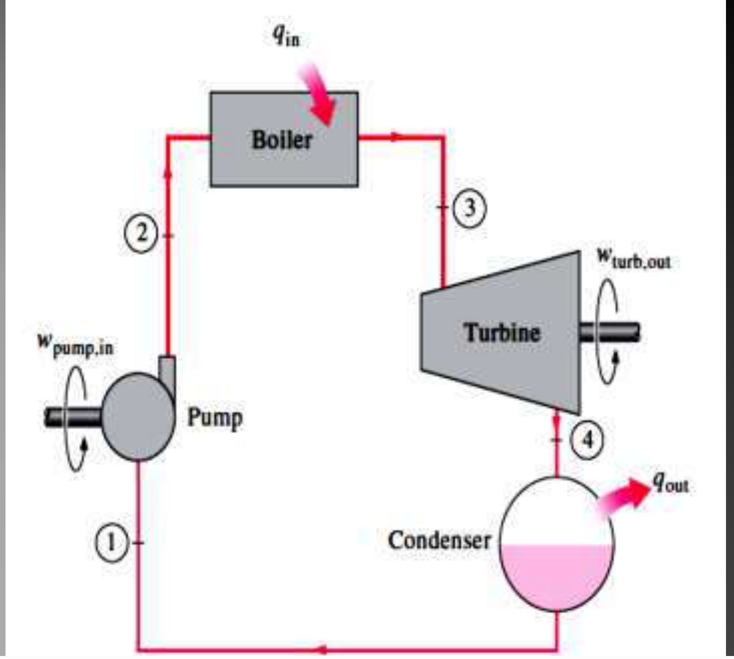


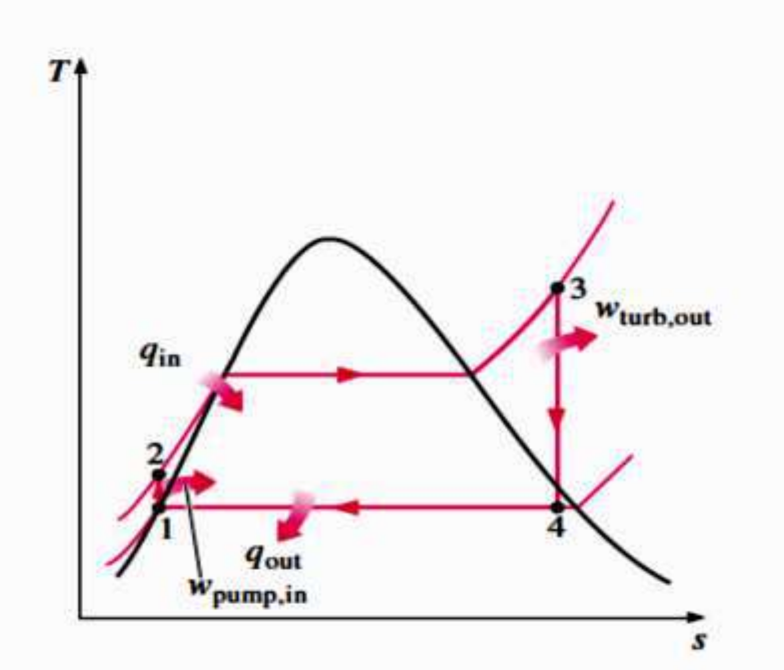
Process 3-4:

Isentropic expansion in a turbine:

The superheated vapor at state 3 enters the turbine, where it expands isentropically and produces work by rotating the shaft connected to the electric-generator.

The pressure and temperature of the steam drop during this process to the values at state 4, where steam enters the condenser.





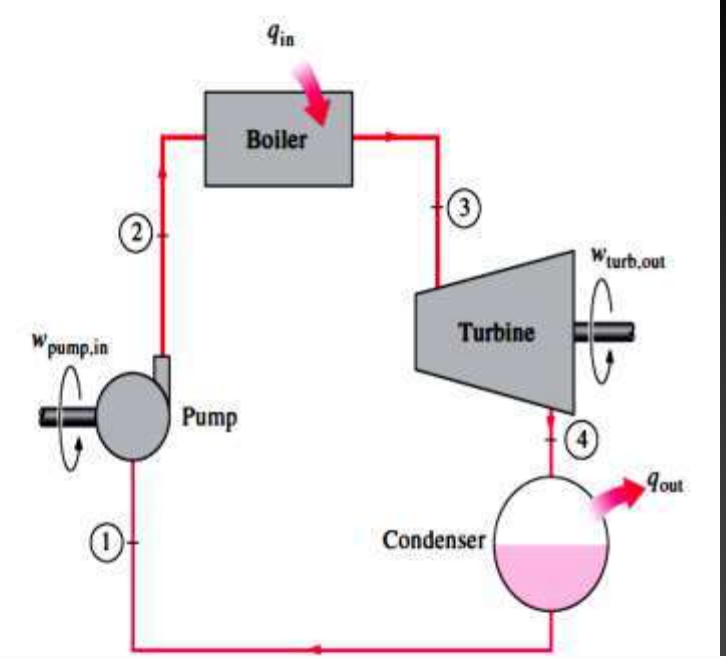
Process 4-1:

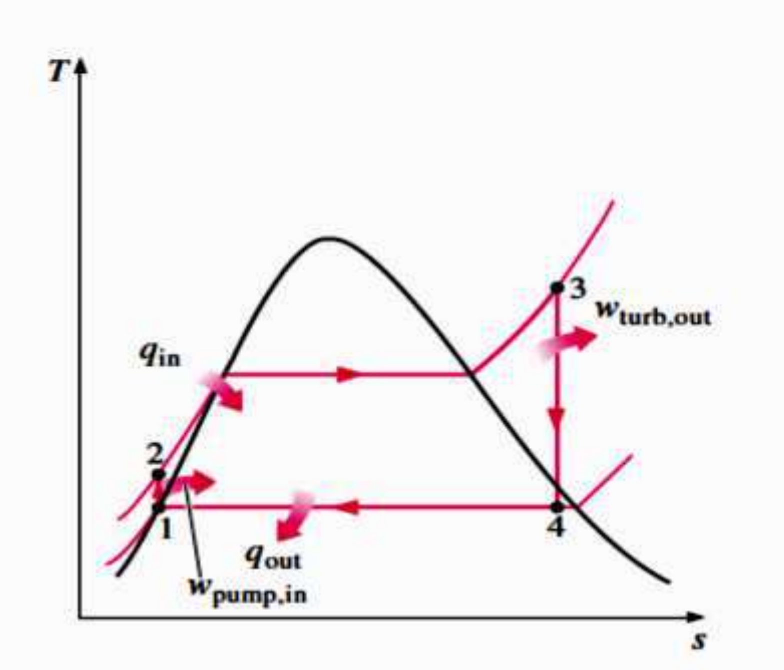
Constant pressure heat rejection in a condenser:

At the state 4 the steam is usually a saturated liquid-vapor mixture with a high quality.

Steam is condensed at constant pressure in the condenser, which is basically a large heat exchanger, by rejecting heat to a cooling medium such as lake, a river, or the atmosphere.

Steam leaves the condenser as saturated liquid and enters the pump, completing the cycle.





Energy analysis of the ideal Rankine Cycle:

All four components associated with the Rankine cycle the pump, boiler, turbine and condenser are steady-flow devices, and thus all four processes that make up the Rankine cycle can be analyzed as steady-flow processes.

The potential and kinetic energy changes of the steam are usually **small** relative to the work and heat transfer terms and are therefore **neglected.**

The steady-flow energy equation per unit mass of the steam reduces to,

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i$$

The boiler and the condenser **do not involve any work**, and the pump and the turbine are assumed to be isentropic.

Then the conservation of energy relation for each device can be expressed as follows:

The pump work during process 1–2 is given by,

$$Tds = dh - vdp$$

Pump handles water which can be assumed to be incompressible.

From property relationship for isentropic process, $ds = 0$

$$dh = vdp$$

therefore, $h_2 - h_1 = v(p_2 - p_1)$

For process 1–2 $\therefore w_{pump,in} = h_2 - h_1 = v(p_2 - p_1)$

The turbine work during the process 3-4 is given by,

$$h_1 = h_{f1} @ pressure\ P_1 \quad and \quad v \approx v_1 = v_f\ @ pressure\ p_1$$

The heat added per unit mass in the boiler during the process 2-3 is given by,

$$q_{in} = h_3 - h_2$$

The turbine work during the process 3-4 is given by

$$w_{turb,out} = h_3 - h_4$$

The heat rejected per unit mass in the condenser during the process 4-1 is given by,

$$q_{out} = h_4 - h_1$$

Where,

$h_2, h_3,$ and h_4 are the enthalpies at state points 2,3, and 4 respectively

Thermal efficiency:

The thermal efficiency of the Rankine cycle is given by,

$$\eta = \frac{Net\ workdone}{Heat\ supplied} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

Or

$$\eta = \frac{w_{turb,out} - w_{pump,in}}{q_{in}} = \frac{(h_3 - h_4) - h_2 - h_1}{(h_3 - h_2)}$$

The pump work is usually very small compared to turbine work.

Hence, sometimes, it is neglected. In that case,

$$\eta = \frac{(h_3 - h_4)}{(h_3 - h_2)}$$

Work ratio:

$$w_R = \frac{Net\ workdone}{Positive\ work} = \frac{w_{net}}{w_{turb,out}} = \frac{(h_3 - h_4) - h_2 - h_1}{(h_3 - h_4)}$$

Steam flow rate:

It is defined as the rate of stem flow in **kg/hr** required to produce unit shaft power output (1kW).

It is a measure of the capacity of a steam power plant.

$$Steam\ rate = \frac{3600}{w_{net}} \ \frac{kg}{kW-hr}, \quad where\ w_{net}\ is\ in \frac{kJ}{kg}.$$

Heat flow rate:

It is the rate of heat input $\mathbf{Q_1}$ in **kJ/hr** required to produce unit power output of 1kW.

Heat rate is an alternative to efficiency.

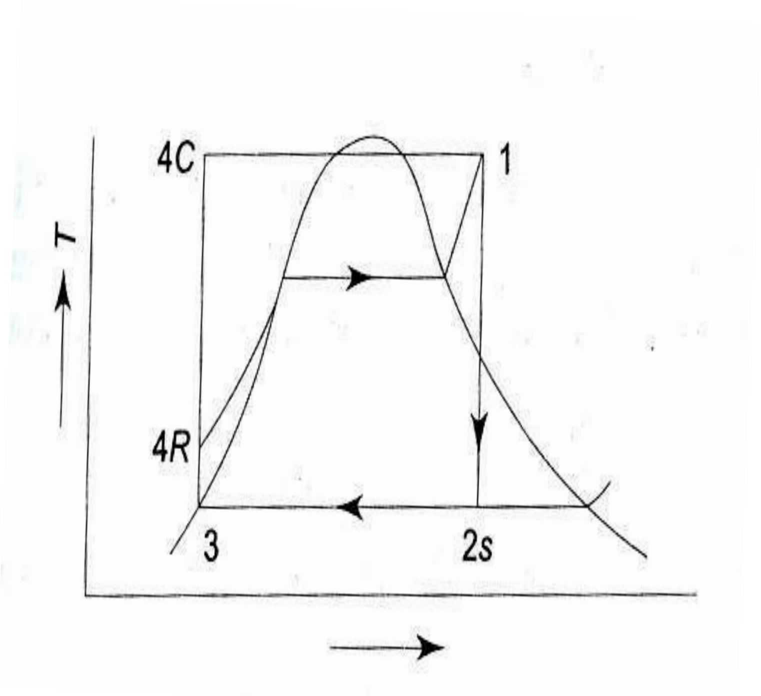
$$Heat\ rate = \frac{3600\ Q_1}{w_{net}} = \frac{3600}{\eta} \frac{kJ}{kW-hr}$$

Comparison between Rankine and Carnot cycle:

a) For the same maximum and minimum temperatures Rankine cycle has lower efficiency than that of the Carnot cycle.

b) For the same maximum and minimum temperatures Rankine cycle has the higher specific output than that of the Carnot cycle.

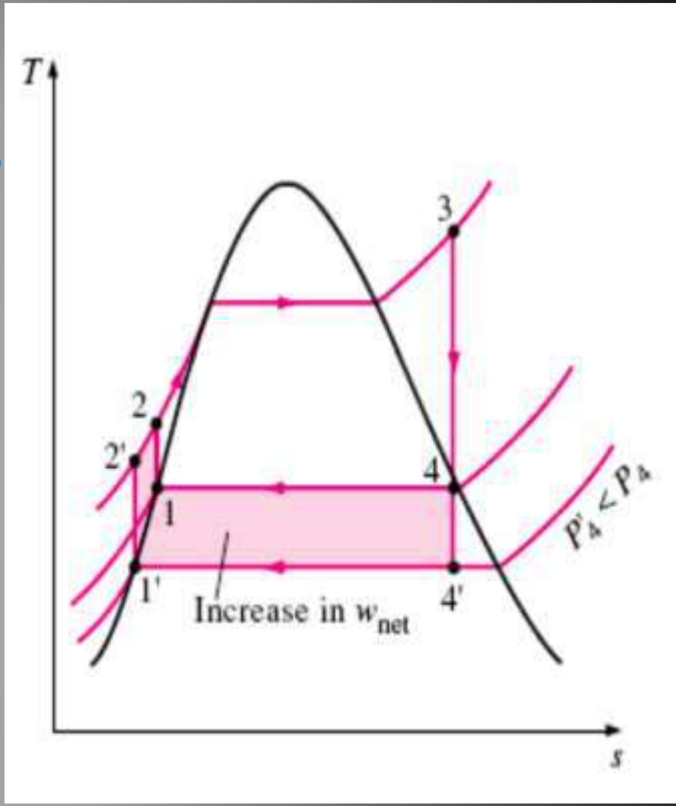
c) Compression of wet vapor is difficult and involves large work in case of Carnot cycle when compared to the pumping work of feed water to the boiler in case of a Rankine cycle.



Effects of pressure and temperature on Rankine cycle performance:

a) Lowering the condenser pressure increases the thermal efficiency of the cycle:

The effect of lowering the condenser pressure on the Rankine cycle efficiency is illustrated on a T-s diagram as in Fig.



For comparison purposes, the turbine inlet state is maintained the same.

The colored area on this diagram represents the increase in net work output as a result of lowering the condenser pressure from P_4 to $P_{4'}$.

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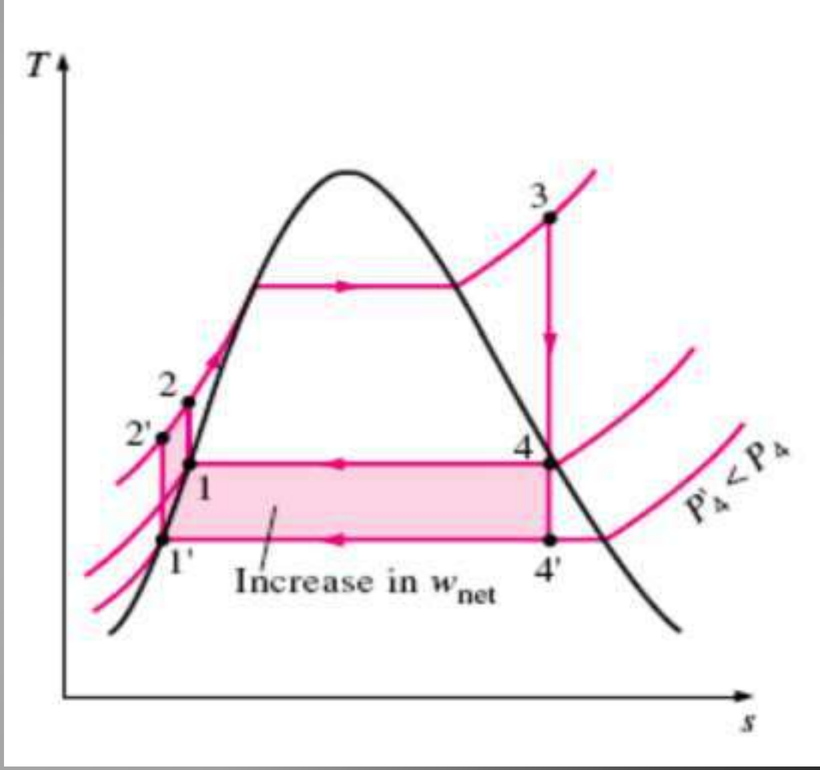
The heat input requirements also increase (are under curve 2-2’), but this increase is very small.

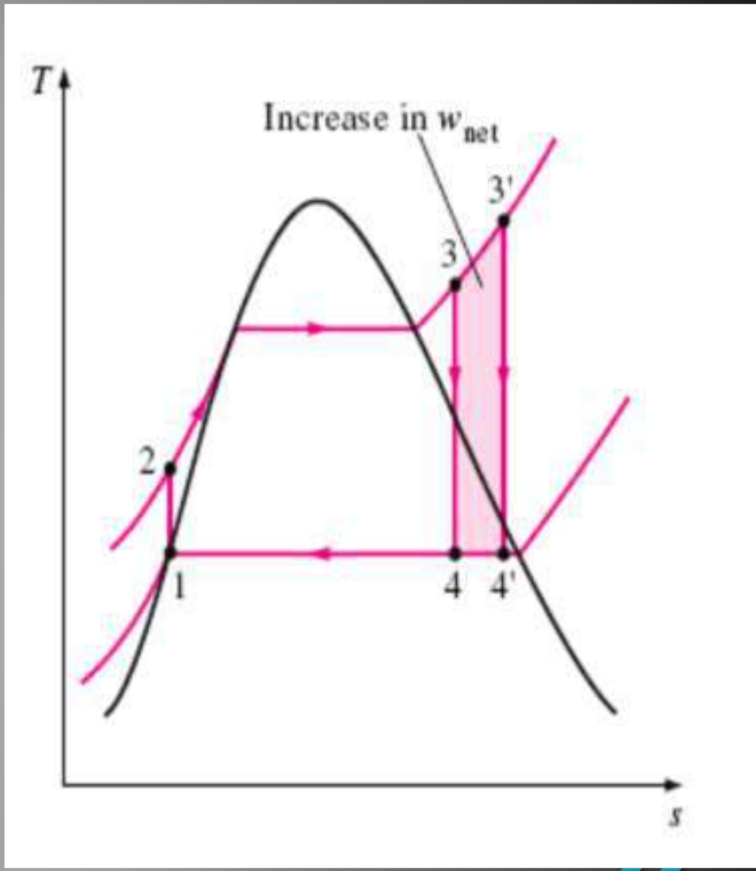
Thus the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.

However the lowest pressure of condenser under ideal conditions is limited to the saturation temperature of the cooling water or air (cooling medium).



b) Superheating the Steam to High temperatures increases thermal efficiency of the cycle:

The effect of superheating on the performance of the vapor power cycles is illustrated on a T-s diagram as in Fig.



The colored area on this diagram represents the increase in the net work.

The total area under the curve 3-3' represents the increase in the heat input.

Thus the net work and heat input increase as a result of superheating the steam to a

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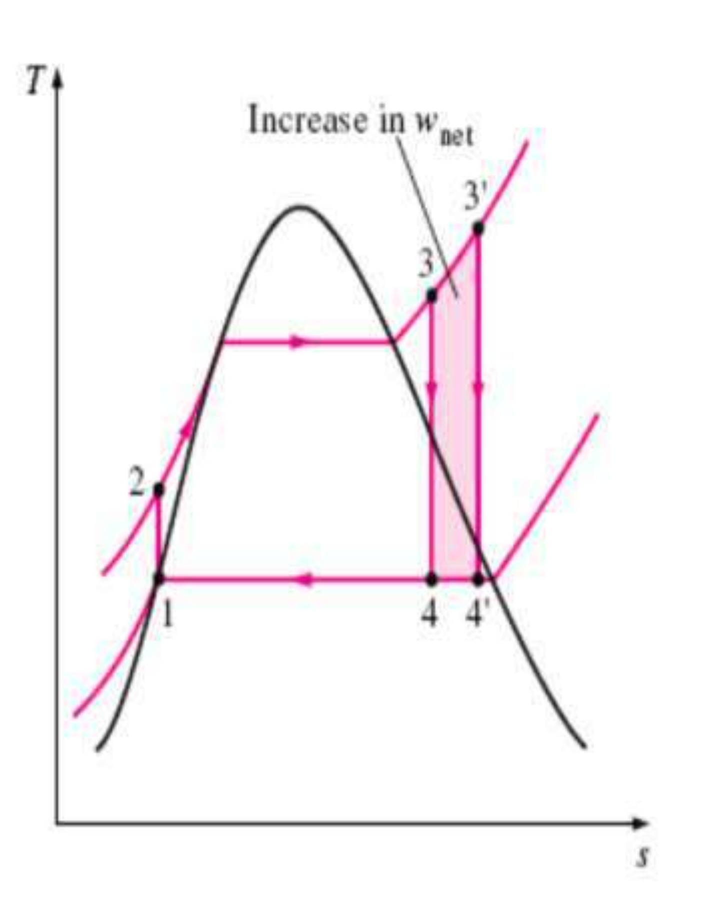
The overall effect is an increase in thermal efficiency, however, since the average temperature at which the heat is added increases.

Superheating the steam to higher temperatures has another very desirable effect:

It decreases the moisture content of the steam at the turbine exit, as can be seen from the T-s diagram.

The temperature to which steam can be superheated is



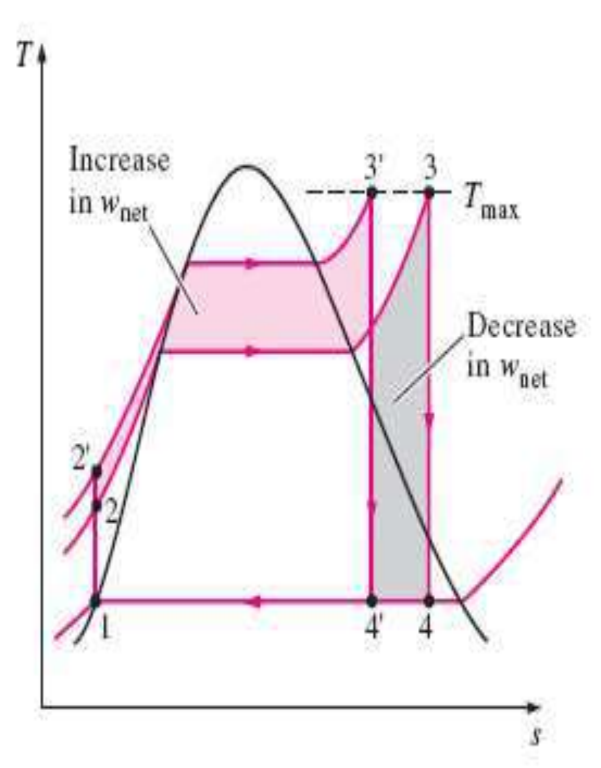
c) Increasing the boiler pressure and temperature increases the efficiency of the cycle:

The increase in the operating pressure of the boiler automatically raises the temperature at which the boiling takes place.

This in turn, raises the average temperature at which heat is transferred to the steam and raises the thermal efficiency of the cycle.

The effect of increasing the boiler pressure on the performance of the vapor power cycles is illustrated on a T-s diagram as in Fig.

Notice that for a fixed turbine inlet temperature, the cycle shifts the left and the moisture content of steam at



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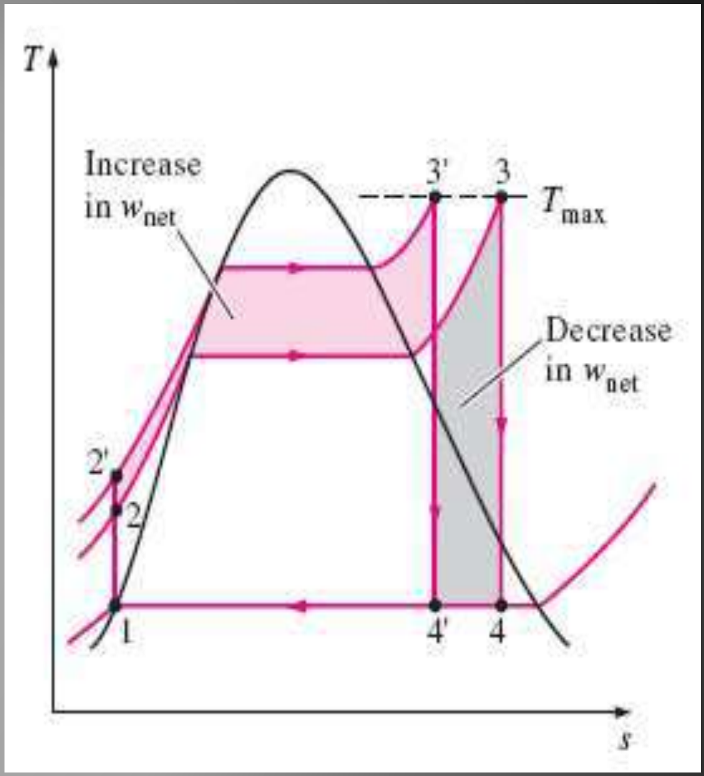
- High moisture content results in erosion of blade surfaces, affecting their life.
- Normally the moisture content at the turbine exhaust **should not exceed 15%.**
- This undesirable side effect can be corrected, however, by reheating the steam.
- Fig. shows the variation of efficiency with boiler pressure.
- Efficiency increases with boiler pressure and reaches maximum value when the pressure is **about 160bar.**
- Further increase in boiler

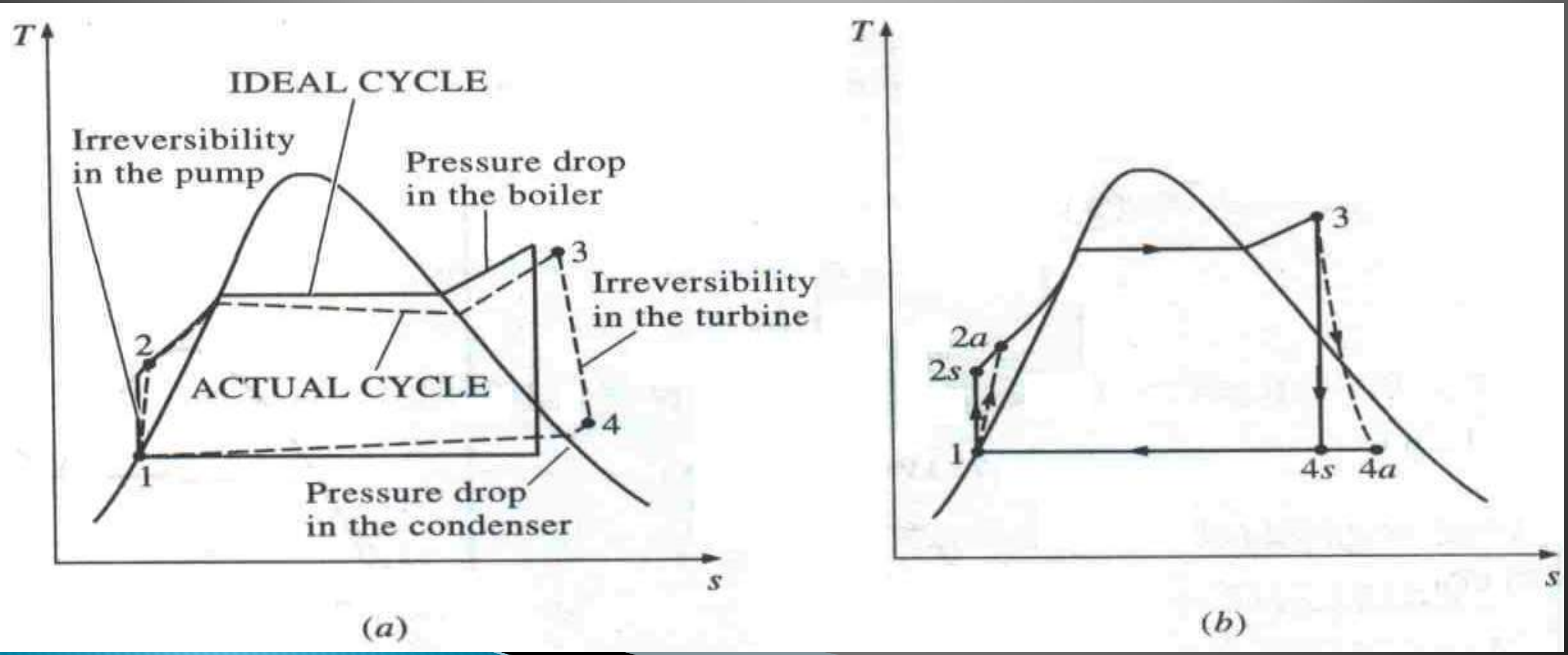


Actual vapor power cycles:

The actual vapor power cycle differs from the ideal Rankine cycle, as illustrated in Fig (a), as a result of irreversibilities in various components. Fluid friction and heat loss to the surroundings are the two common sources of the irreversibilities.

Fluid friction causes pressure drops in the boiler, the condenser and the piping between various components.

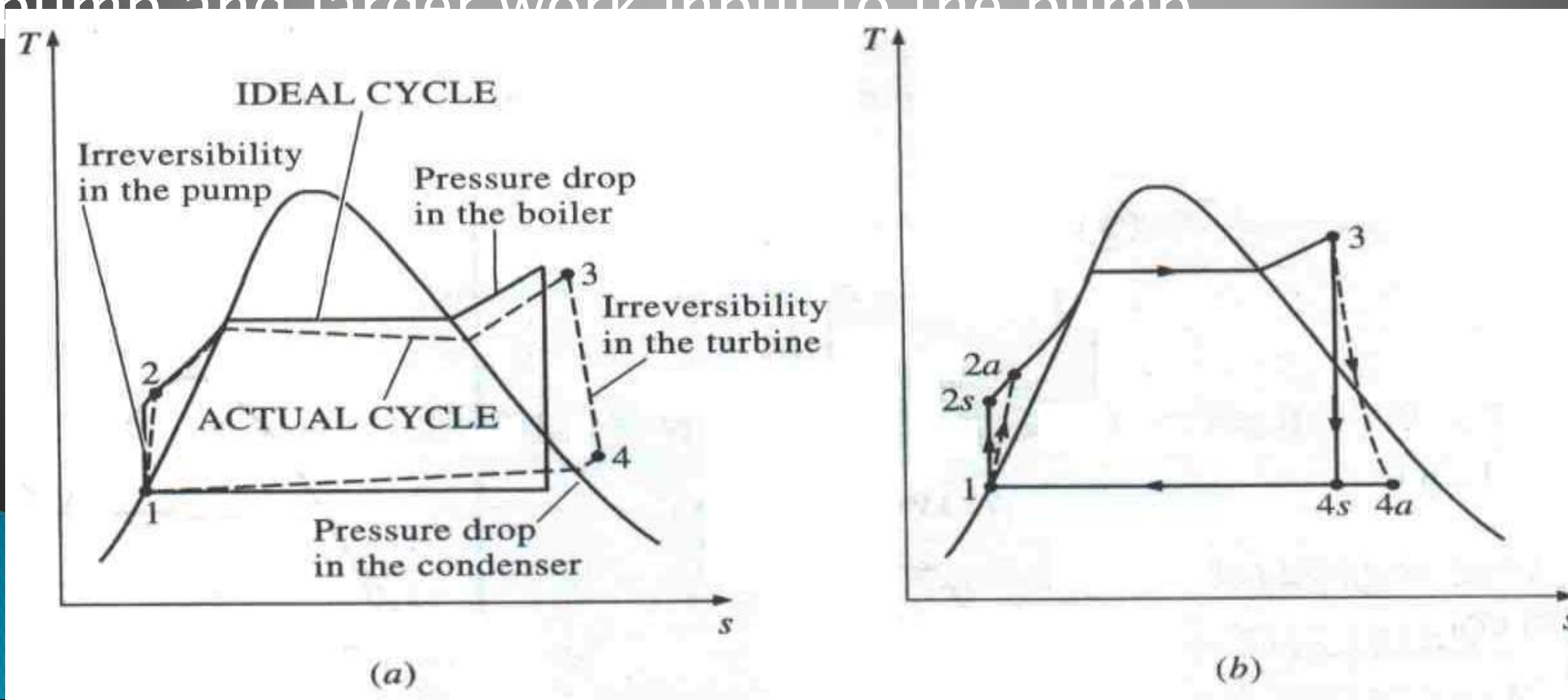
As a result, steam leaves the boiler at somewhat lower



Actual vapor power cycles:

Also, the pressure at the turbine inlet is somewhat lower than that in the boiler exit due to the pressure drop in the connecting pipes.

The pressure drop in the condenser is usually very small. To compensate for these pressure drops, the water must be pumped to a sufficiently higher pressure than the ideal cycle calls for. This requires a larger pump and larger work input to the pump.

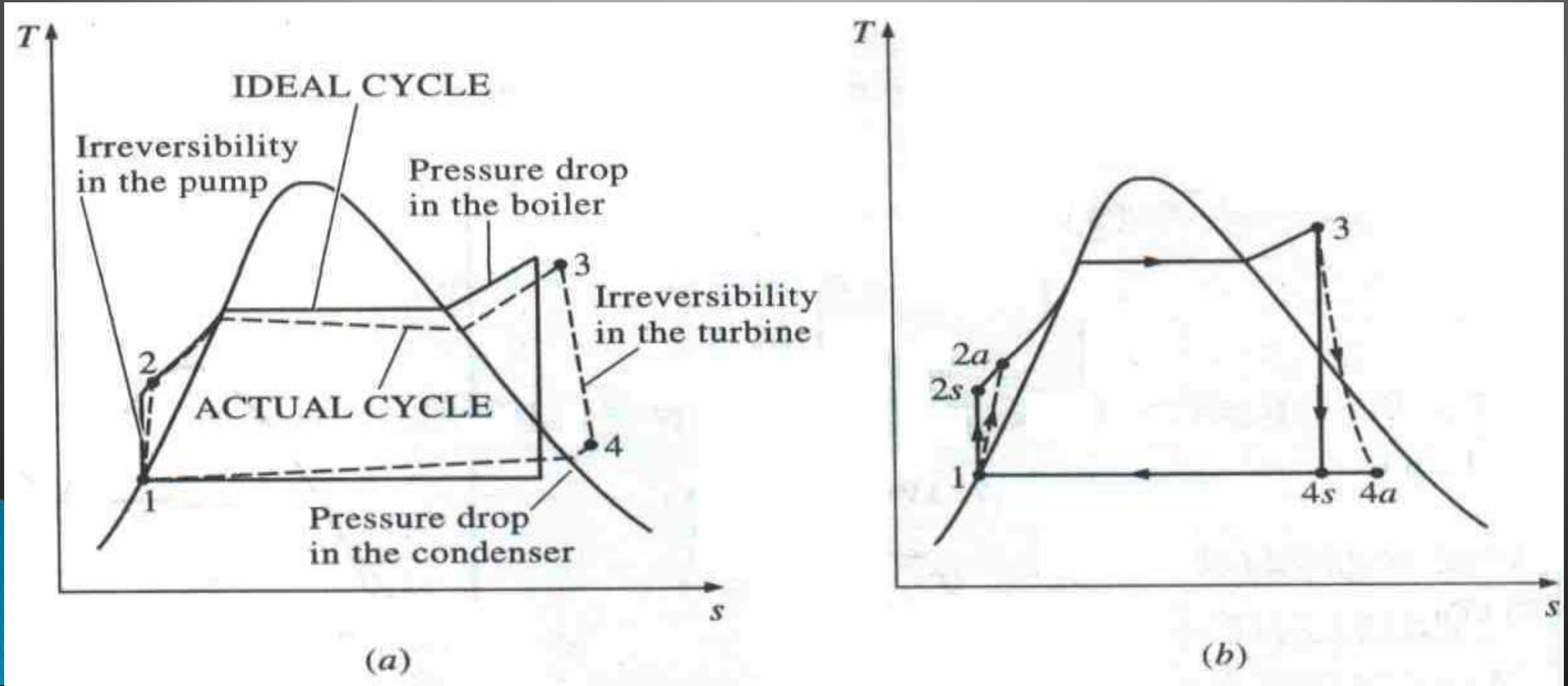


Actual vapor power cycles:

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components.

To maintain the same level of net work output, more heat needs be transferred to the steam in the boiler to compensate for these undesirable heat losses.

As a result the cycle efficiency decreases.



Under ideal conditions, the flow through pump and turbine is isentropic.

The deviation of the actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as,

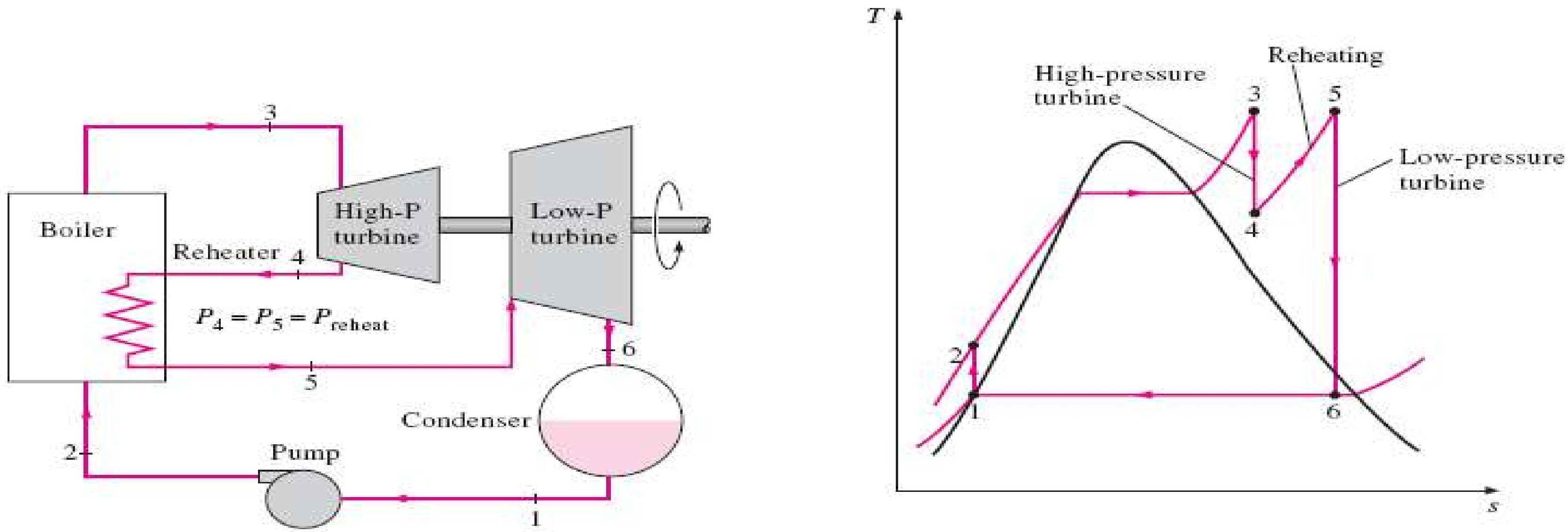
$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

And

$$\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

Where states 2a and 4a are the actual exit states of the pump and the turbine, respectively, and 2s and 4s are the corresponding states for the isentropic case (Fig b).

The ideal reheat Rankine cycle:

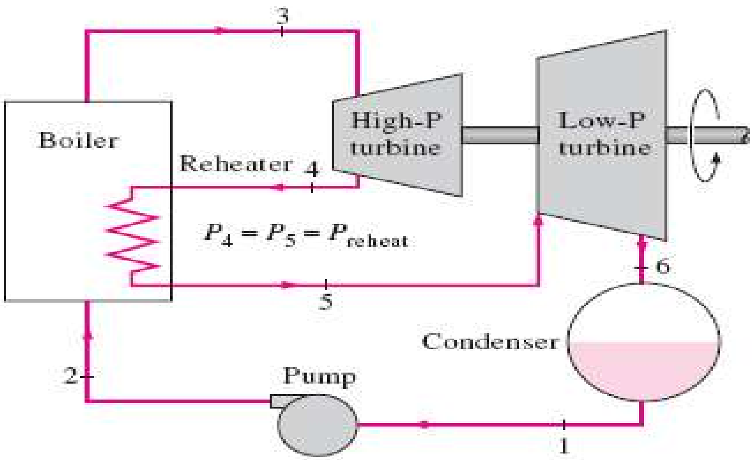


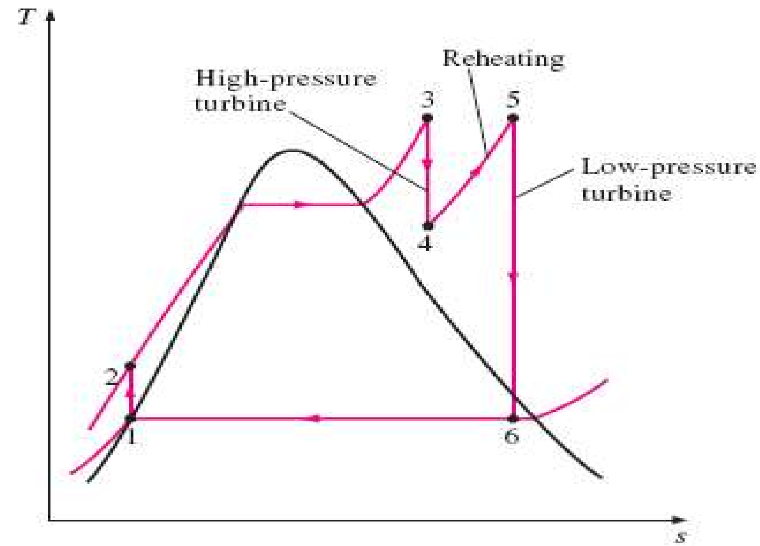
The T-s diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Fig.

The ideal reheat Rankine cycle differs from the simple ideal Rankine cycle in that the expansion process takes place in two stages.

In the first stage (HP turbine), steam is expanded isentropically to an intermediate pressure and sent back to the boiler where it is reheated at constant

The ideal reheat Rankine cycle:





Steam then expands isentropically in the second stage (LP turbine) to the condenser pressure.
Thus the total heat input, the heat rejected, the total turbine work output and total pump work for a reheat cycle become,

$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4)$$

$$q_{out} = (h_6 - h_1) = (h_6 - h_{f1})$$

And

$$w_{turb,out} = w_{turb,I} + w_{turb,II} = (h_3 - h_4) + (h_5 - h_6)$$

$$w_{pump,in} = (h_2 - h_1) = (h_{f2} - h_{f1}) = v_1(p_2 - p_1)$$

The net work done per kg of steam,

$$w_{net} = (h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)$$

Therefore thermal efficiency,

$$\eta = \frac{Net\ Workdone}{Heat\ supplied} = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

The incorporation of the single reheat in a modern power plant improves the cycle efficiency by 4 to 5% by increasing the average temperature at which heat is transferred to the steam.

The ideal Regenerative Rankine cycle:

Reheating has the limited ability to improve the thermodynamic efficiency of Rankine cycle, but is quite useful in the reduction of moisture in the turbine.

However it is observed that the largest single loss of energy in a power plant occurs at the condenser in which the heat is rejected to the coolant.

Hence reducing this rejected heat drastically improves efficiency.

In both ideal and reheat cycle the condensate is returned to the boiler at the lowest temperature of the cycle.

The fluid is heated to saturation by direct mixing in the steam drum of the boiler, by furnace radiation in both tubes or by gas convection heating by the flue gases in the economizer.

All these methods involve large temperature differences and are inherently irreversible.

Instead of resorting to such a procedure, a method of feed water heating is considered.

In the simple Rankine cycle the average temperature of heat addition is quite low.

If the amount of heat required for this purpose is supplied internally, the cycle thermal efficiency would approach to that of Carnot cycle.

This could be done in a regenerative cycle in which feed water is preheated by the expanding steam.

The thermal efficiency of the Rankine cycle increases as a result of regeneration.

This is because regeneration raises the average temperature at which heat is transferred to the steam in the boiler by raising the temperature of water before it enters the boiler.

A practical regeneration process in steam power plants is accomplished by extracting or “bleeding” steam from the turbine at various points.

This steam which could have produced more work by expanding further in the turbine is used to heat the feed water instead.

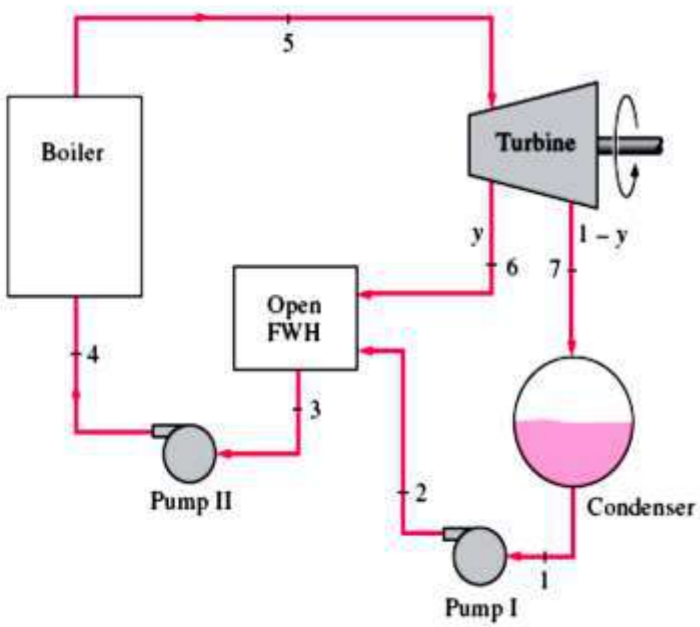
The device where the feed water is heated by regeneration is called **regenerator** or a **feed water heater (FWH)**.

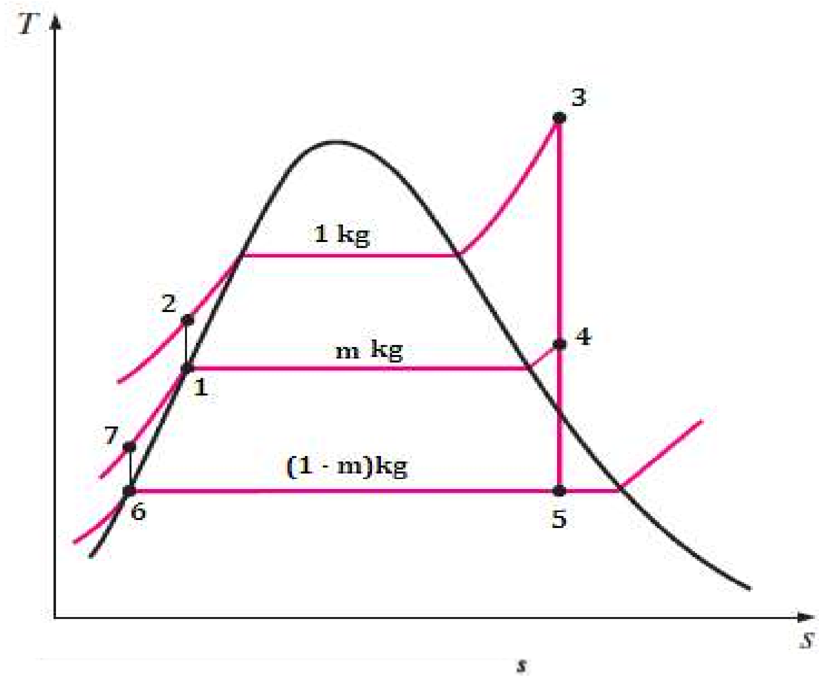
A feed water heater is basically a heat exchanger where heat is transferred from steam to the feed water either by mixing the two fluid streams (open feed water heaters) or without mixing them (closed feed water heaters).

Open feed water heaters:

An open (or direct-contact) feed water heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feed water exiting the pump.

Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure. The schematic of a steam power plant with one open feed water heater (also called single-stage regenerative cycle) and the T-s diagram of the cycle.

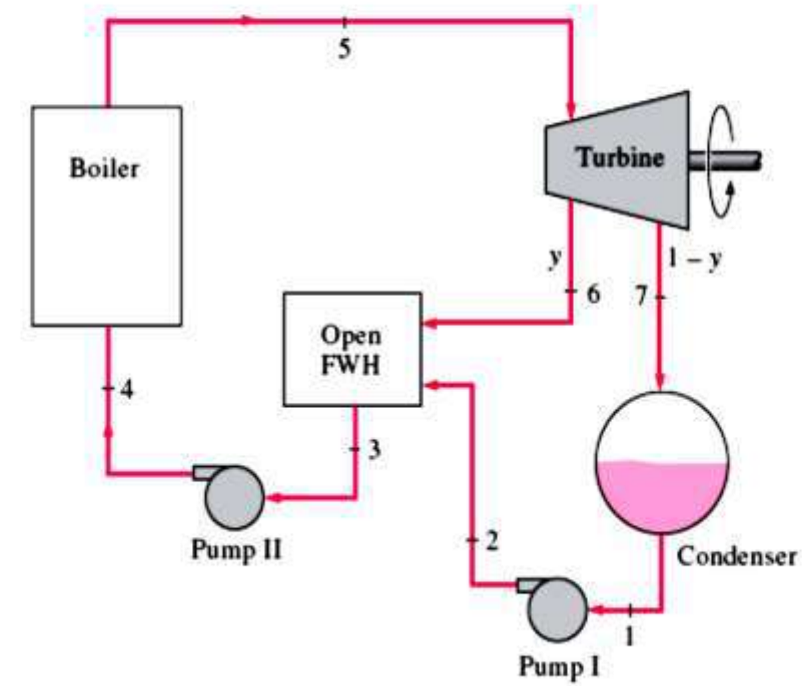


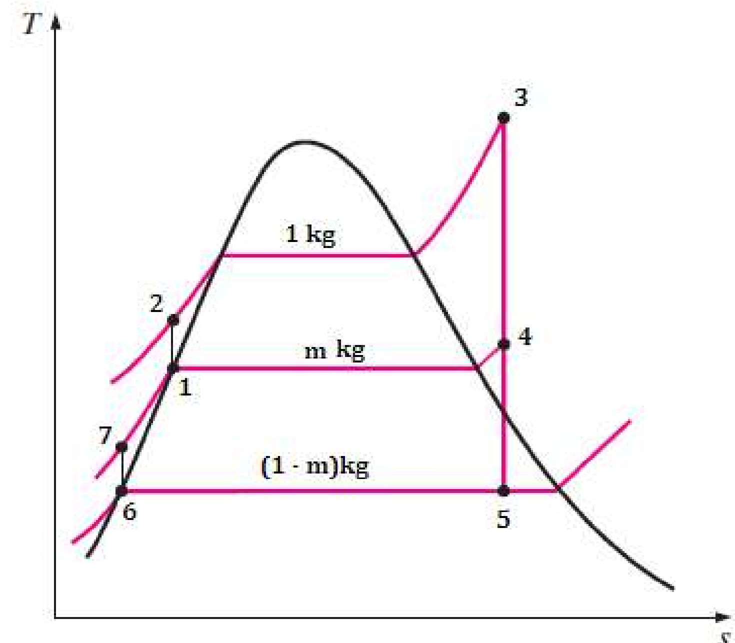


In an ideal regenerative Rankine cycle, steam enters the turbine at the boiler pressure (state 5) and expands isentropically to an intermediate pressure (state 6).

Some steam is extracted at this state and routed to the feed water heater, while the remaining steam continues to expand isentropically to the condenser pressure (state 7).

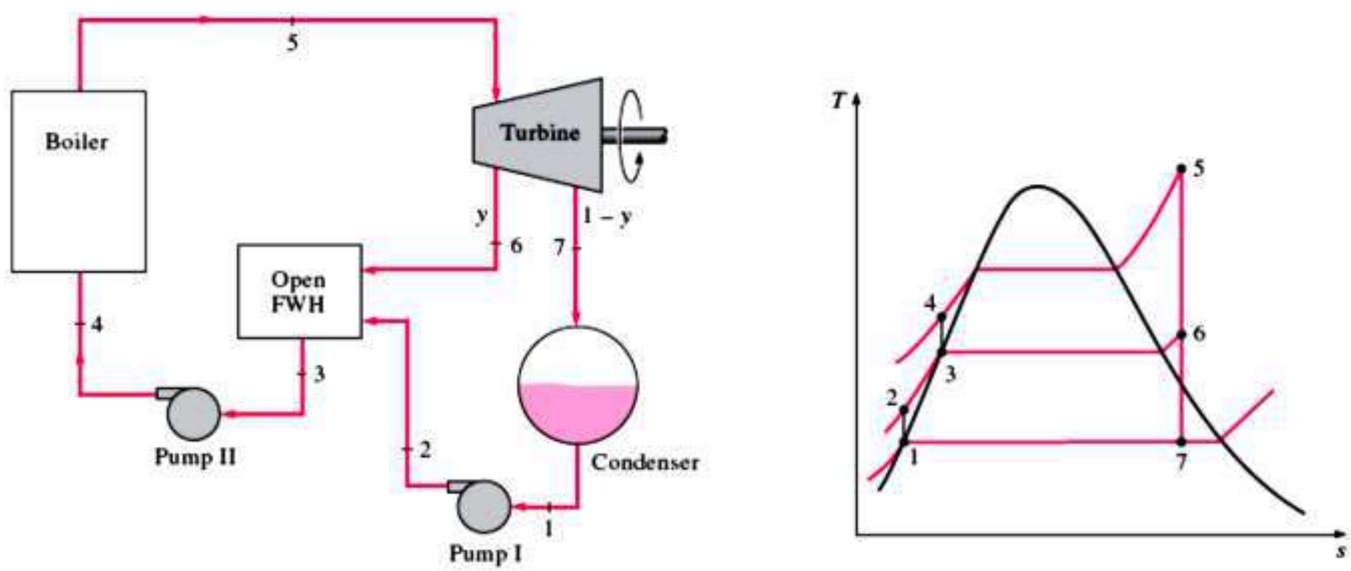
This steam leaves the condenser as a saturated liquid at the condenser pressure (state 1).





The condensed water which is also called the feed water then enters an isentropic pump, where it is compressed to the feed water heater pressure (state 2) and is routed to the feed water heater, where it mixes with the steam extracted from the turbine. The fraction of the steam extracted is such that the mixture leaves the heater as a saturated liquid at the heater pressure (state 3).

A second pump raises the pressure of the water to the boiler pressure (state 4). The cycle is completed by heatin tate 5).



Let for each kg of steam leaving the boiler, y kg expands partially in the turbine and is extracted at state 6.

The remaining (1 – y) kg expands completely to the condenser pressure. The heat and work interactions of a regenerative Rankine cycle with one feed water heater can be expressed per unit mass of steam flowing through the boiler as follows:

$$q_{in} = h_5 - h_4 \qquad \& \qquad q_{out} = (1-y)(h_7 - h_1)$$

$$w_{turb,out} = (h_5 - h_6) + (1-y)(h_6 - h_7)$$

$$w_{pump,in} = (1-y)w_{pumpI,in} + w_{pumpII,in}$$

Where, $y = fraction\ of\ steam\ extracted = \dot{m}_6/\dot{m}_5$

$$w_{pumpI,in} = v_1(P_2 - P_1) \qquad \& \qquad w_{pumpII,in} = v_3(P_4 - P_3)$$

Thermal efficiency of the cycle is given by:

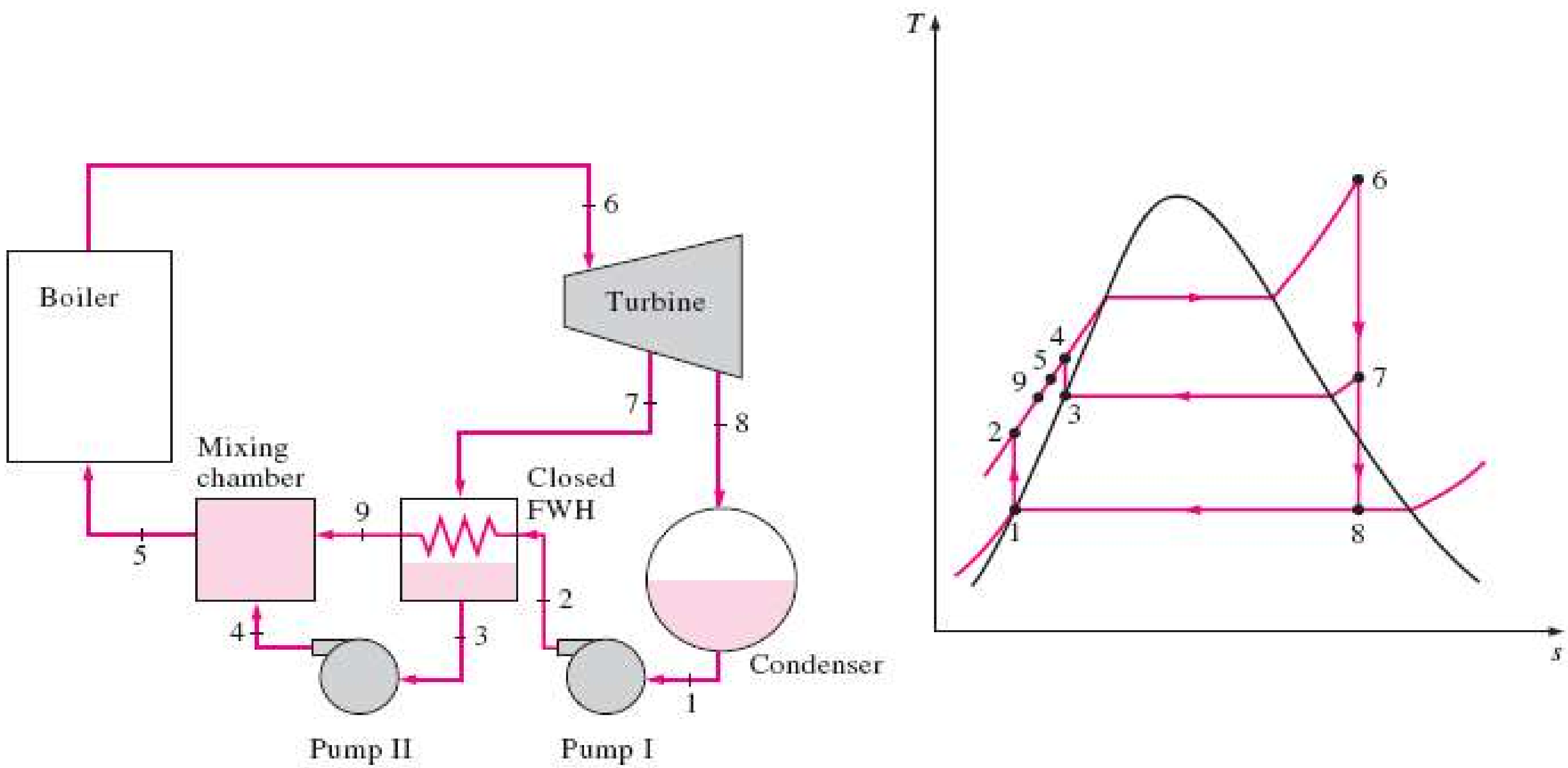
$$\eta = \frac{Net\ Workdone}{Heat\ Supplied} = \frac{q_{in} - q_{out}}{q_{in}} = \frac{(h_5 - h_4) - [(1-y)(h_7 - h_1)]}{(h_5 - h_4)}$$

Closed feed water heaters:

In this heat is transferred from the extracted steam to the feed water without any mixing taking place.

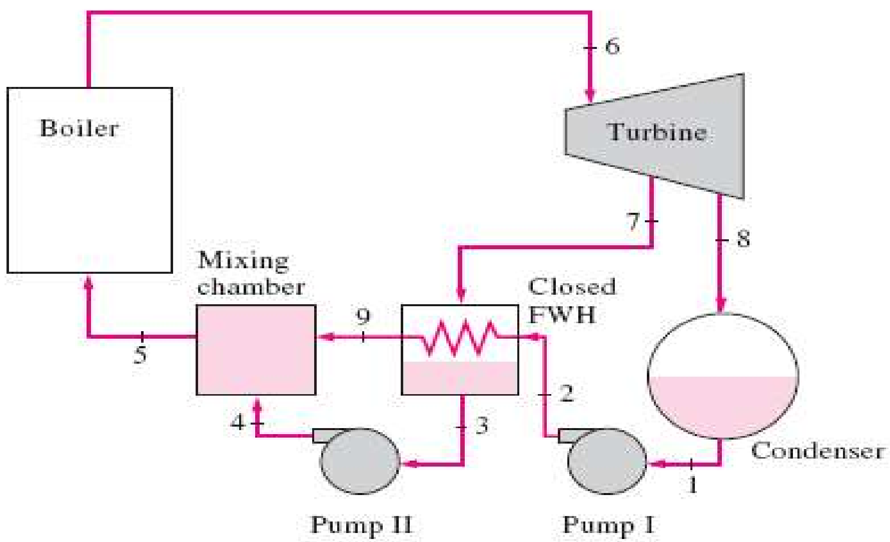
The two streams now can be at different pressures, since they do not mix.

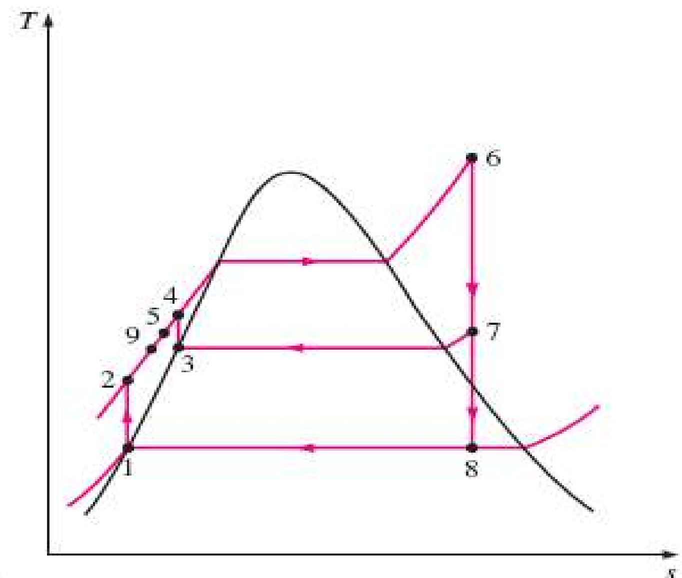
The schematic of a power plant with one closed feed water heater and the T-s diagram of the cycle are



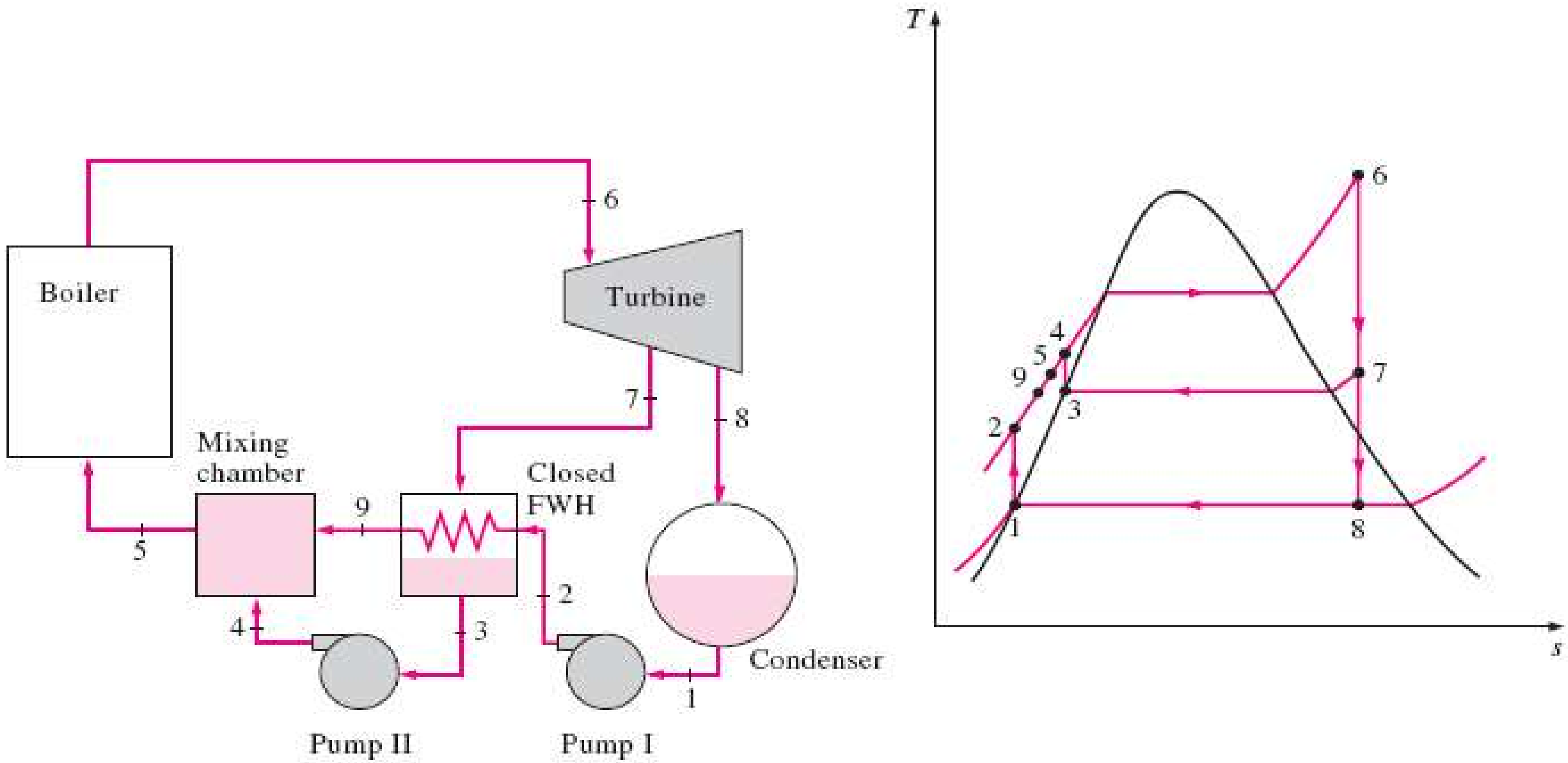
In an ideal closed feed water is heater, the feed water is heated is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure.

In actual power plants, the feed water leaves the heater below the exit temperature of the extracted steam because a temperature difference of at least a few degrees is required for any effective heat to take place





- The condensed steam is then either pumped to the feed water heater line or routed to another heater or to the condenser through a device called trap.
- A trap allows the liquid to be throttled to a lower pressure region but traps the vapor.
- The enthalpy of the steam remains constant during this throttling process.



Regenerative plant with two open feed water heaters:

Based on a total flow of 1kg of steam,

Turbine output,

$$W_T = 1 \times (h_7 - h_8) + (1 - m_1)(h_8 - h_9) + (1 - m_1 - m_2)(h_9 - h_{10})$$

Pump work,

$$W_P = (1 - m_1 - m_2)(h_2 - h_1) + (1 - m_1)(h_4 - h_3) + 1 \times (h_6 - h_5)$$

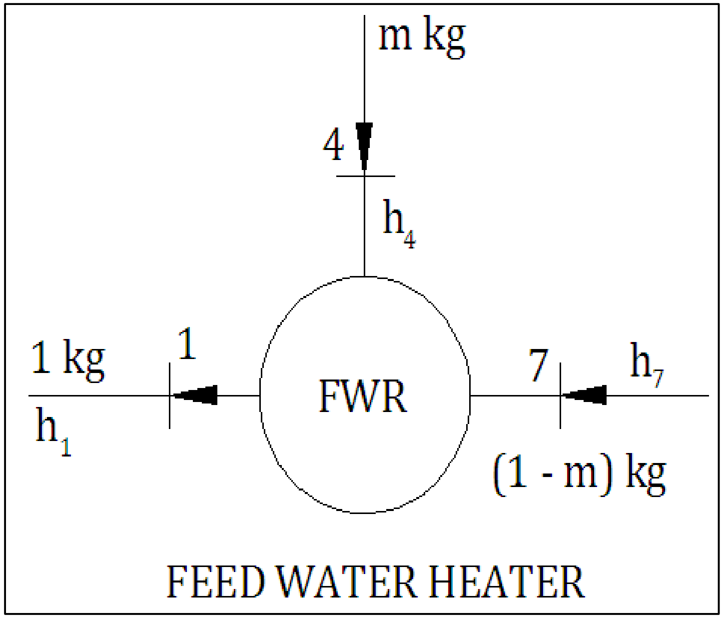
$$W_{net} = W_T - W_P$$

Heat added from external source,

$$Q_1 = 1 \times (h_7 - h_6)$$

Therefore,

$$Cycle\ efficiency, \quad \eta = \frac{W_{net}}{Q_1}$$



Regenerative plant with 2 closed feed water heaters:

On the basis of a total flow of 1kg of steam,

Turbine output,

$$W_T = 1 \times (h_7 - h_8) + (1 - m_1)(h_8 - h_9) + (1 - m_1 - m_2)(h_9 - h_{10})$$

Pump work, $W_P = (h_2 - h_1)$

$$W_{net} = W_T - W_P$$

Heat added from external source,

$$Q_1 = (h_7 - h_5)$$

Therefore,

$$Cycle\ efficiency, \quad \eta = \frac{W_{net}}{Q_1}$$

To calculate m_1 and m_2

With open feed water heaters:- applying law of conservation of energy to each heater,

$$total\ energy\ in = total\ energy\ out$$

$$for\ the\ H.P\ heater,\quad m_1 h_8 + (1 - m_1) h_4 = 1 \times h_5$$

$$m_1 (h_8 - h_4) = h_5 - h_4 \quad \Rightarrow \quad m_1 = \frac{h_5 - h_4}{h_8 - h_4}$$

$$for\ the\ L.P\ heater,\quad m_2 h_9 + (1 - m_1 - m_2) h_2 = (1 - m_1) \times h_3$$

$$m_2 (h_9 - h_2) = (1 - m_1)(h_3 - h_2)$$

$$\Rightarrow \quad m_2 = \frac{(h_3 - h_2)}{(h_9 - h_2)}$$

ii) With closed feed water heaters,

$$total\ energy\ in = total\ energy\ out$$

$$for\ the\ H.P\ heater, \quad m_1 h_8 + h_{43} = h_5 + m_1 h_4$$

$$m_1(h_8 - h_4) = h_5 - h_3 \Rightarrow m_1 = \frac{h_5 - h_3}{h_8 - h_4}$$

$$for\ the\ L.P\ heater, \quad m_2 h_9 + h_2 + m_1 h_4 = h_3 + (m_1 + m_2) h_6$$

$$m_2(h_9 - h_6) = (h_3 - h_2) + m_1(h_6 - h_4)$$

$$\Rightarrow m_2 = \frac{(h_3 - h_2) + m_1(h_6 - h_4)}{(h_9 - h_2)}$$

Reheat - Regenerative cycle:

In this plant regeneration as well as multi-stage expansion with reheating, are incorporated.

A schematic flow diagram and T-s diagram of a reheat-regenerative plant with two open feed water heaters are shown below.

Reheating is normally done inside the furnace, in a region where the temperatures are high. Steam extraction for purposes of regeneration can be carried out at any intermediate stage in either of the turbines.

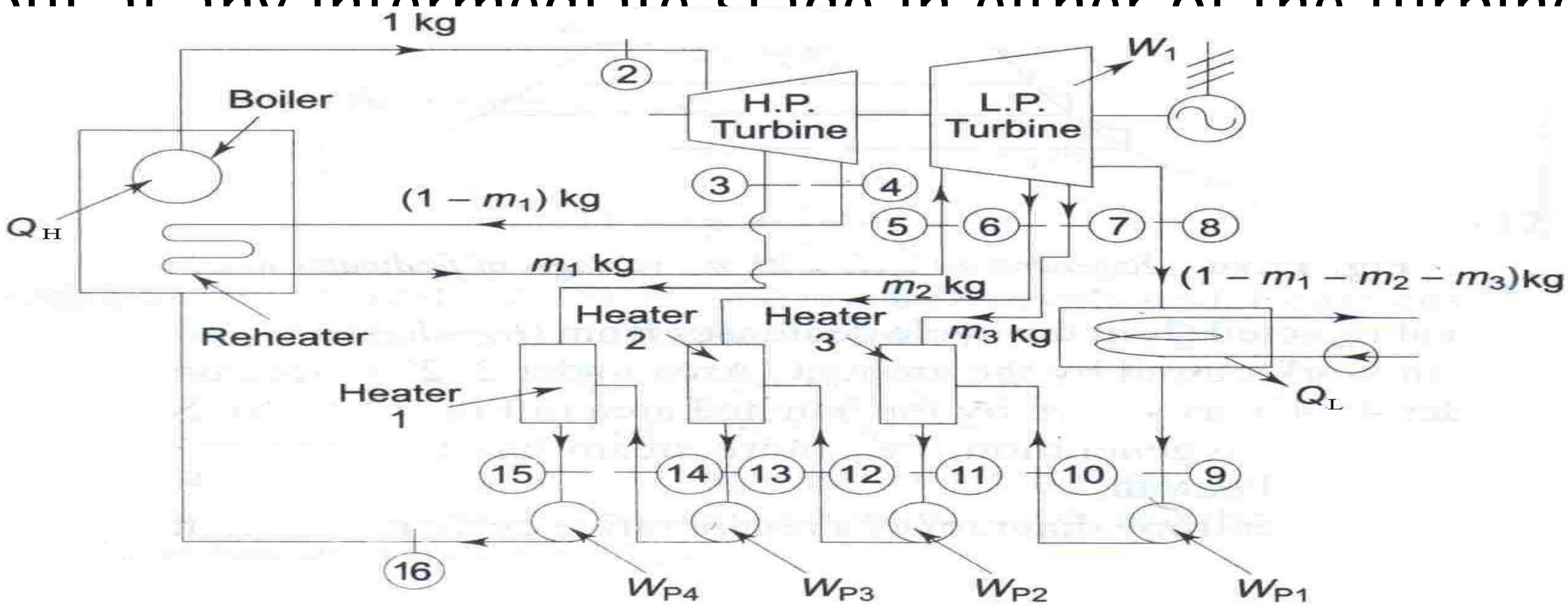


Fig. *Reheat-regenerative cycle flow diagram*

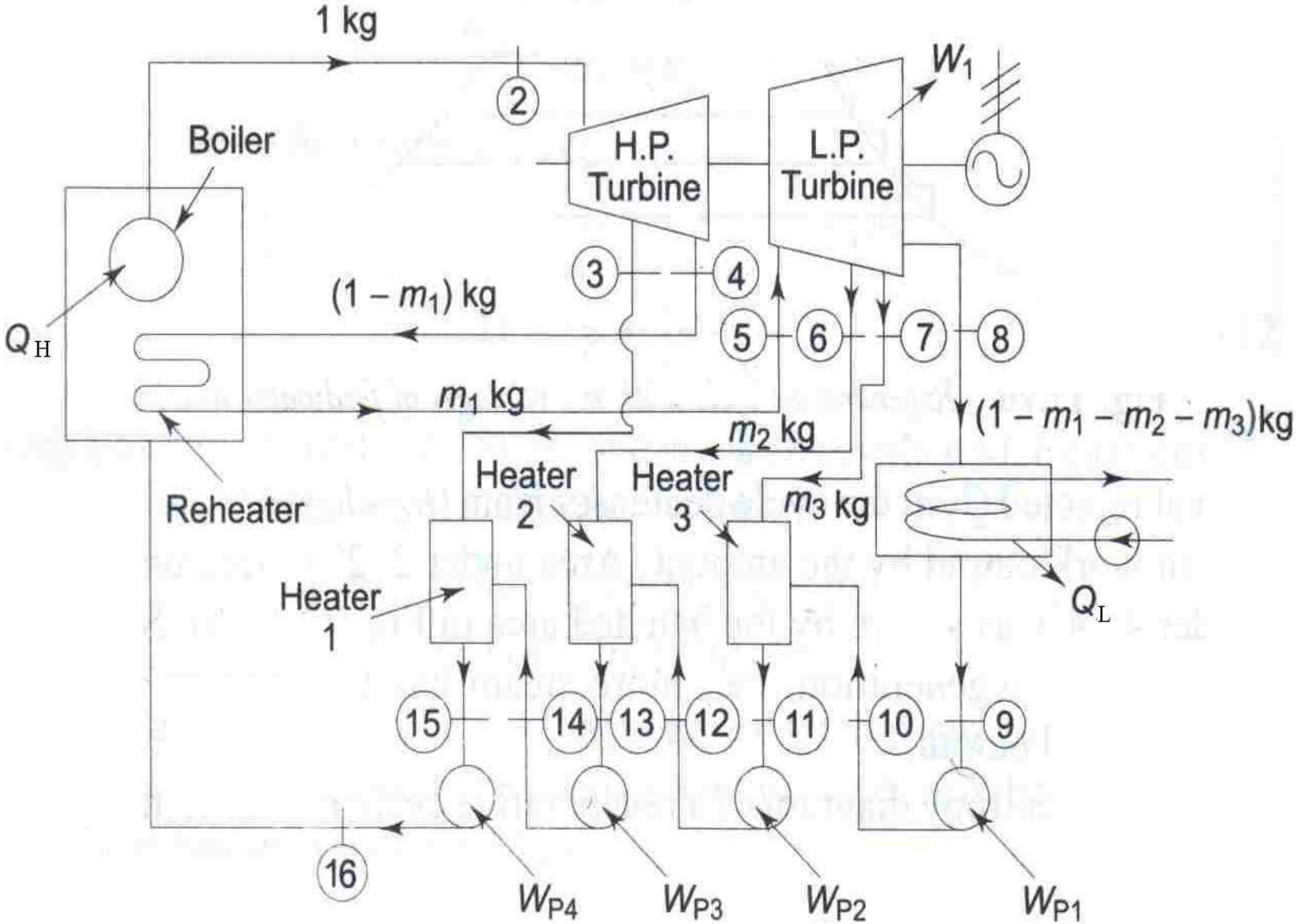


Fig. *Reheat-regenerative cycle flow diagram*

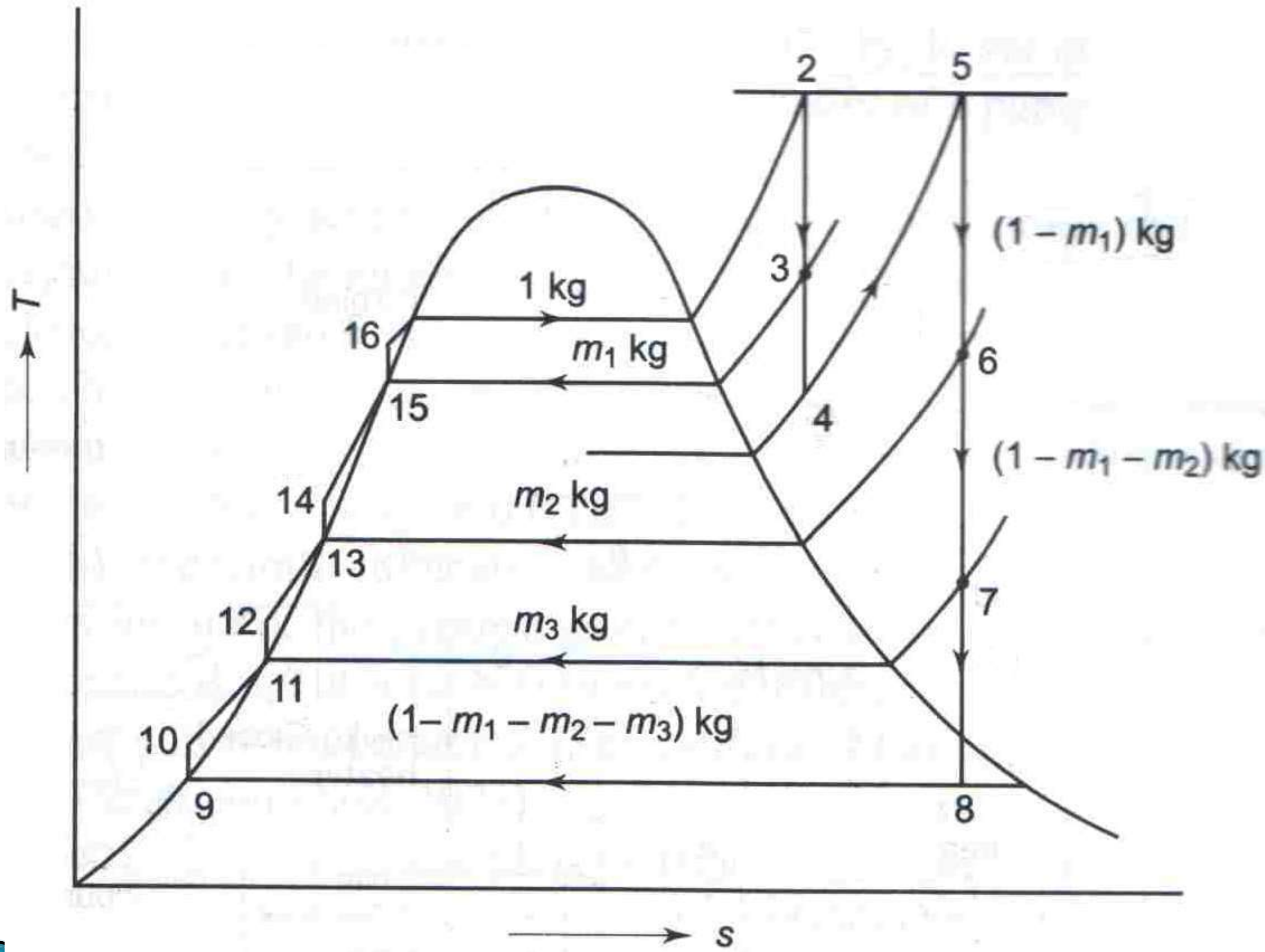


Fig. *T–s diagram of reheat-regenerative cycle*

On the basis of a total flow of 1kg of steam,
Turbine output,

$$W_T = 1 \times (h_7 - h_8) + (1 - m_1)(h_8 - h_9) + (1 - m_1)(h_{10} - h_{11}) + (1 - m_1 - m_2)(h_{11} - h_{12})$$

Pump work,

$$W_P = (1 - m_1 - m_2)(h_2 - h_1) + (1 - m_1)(h_4 - h_3) + 1 \times (h_6 - h_5)$$

$$W_{net} = W_T - W_P$$

Heat added from external source,

$$Q_1 = 1 \times (h_7 - h_6) + (1 - m_1)(h_{10} - h_9)$$

Therefore,

$$Cycle\ efficiency, \quad \eta = \frac{W_{net}}{Q_1}$$

Problems on vapour power cycle

1. **Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: $P_1 = 17.5$ bar $P_2 = 0.75$ bar

$\eta_{Carnot} = ?$ $\eta_{Rankine} = ?$

a) Carnot cycle: At pressure 17.5 bar from steam tables A –14 (**Dry saturated steam**),

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For P = 17.5 bar, using linear interpolation

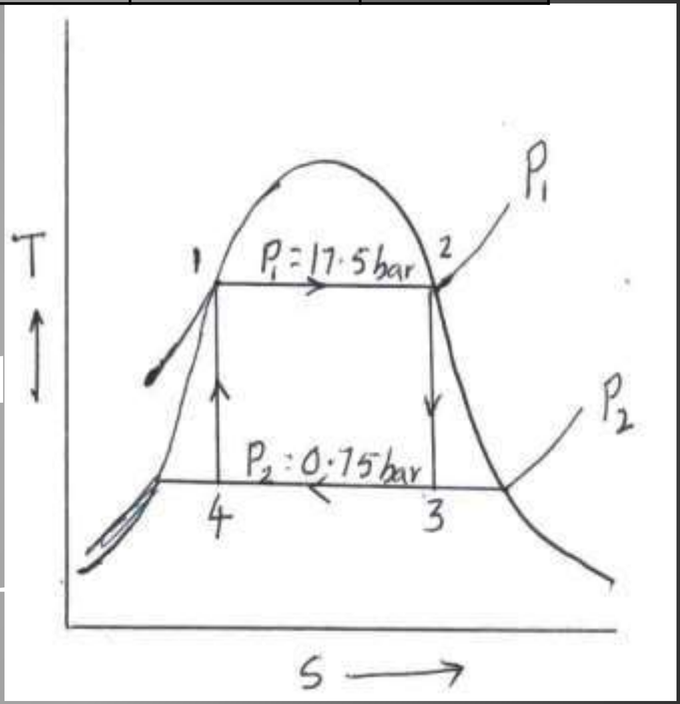
For t_S, = 478.71 K

Similarly, h_f = 878.15 kJ/kg

h_{fg} = 1915.95 kJ/kg h_g = 2794.1 kJ/kg

S_f = 2.3844 kJ/kg^{0}K S_{fg} = 4.0011 kJ/kg^{0}K

S_g = 6.3855 kJ/kg K



Also at pressure 0.75 bar from steam tables (**Wet steam)**

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

$\therefore$ For 0.75 bar, using linear interpolation,

$t_S = 91.74^0C$ $\quad$ $h_f = 384.25$ $\quad$ $h_{fg} = 2278.65$ $\quad$ $h_g = 2662.95$

$S_f = 1.2126$ $\quad$ $S_{fg} = 6.2453$ $\quad$ $S_g = 7.4578$

The Carnot cycle η, η_C =

$$\frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$$

Steam rate or SSC
(specific steam consumption) = $\frac{1}{\oint \delta W} = \frac{1}{W_T - W_P}$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453)$ $\therefore x_3 = 0.828$

∴ Enthalpy at state 3,

$h_3 = h_{f3} + x_3 h_{fg} = 384.25 + 0.828\ (2278.65) = 2271.63$ kJ/kg

∴ Turbine work or expansion work or positive work

$= h_2 - h_3 = 2794.1 - 2271.63 = 522.47$ kJ/kg

Again since the compression process is isentropic

i.e., $S_4 = S_1 = S_{f1} = 2.3844$

Hence $2.3844 = S_{f4} + x_4\ S_{fg4}$

$= 1.2126 + x_4\ (6.2453)\ \therefore x_4 = 0.188$

$\therefore$ Enthalpy at state 4 is $h_4 = h_{f4} + x_4\, h_{fg4}$

$= 384.25 + 0.188\,(2278.65) = 811.79$ kJ/kg

$\therefore$ Compression work, $= h_1 - h_4 = 878.15 - 811.79$

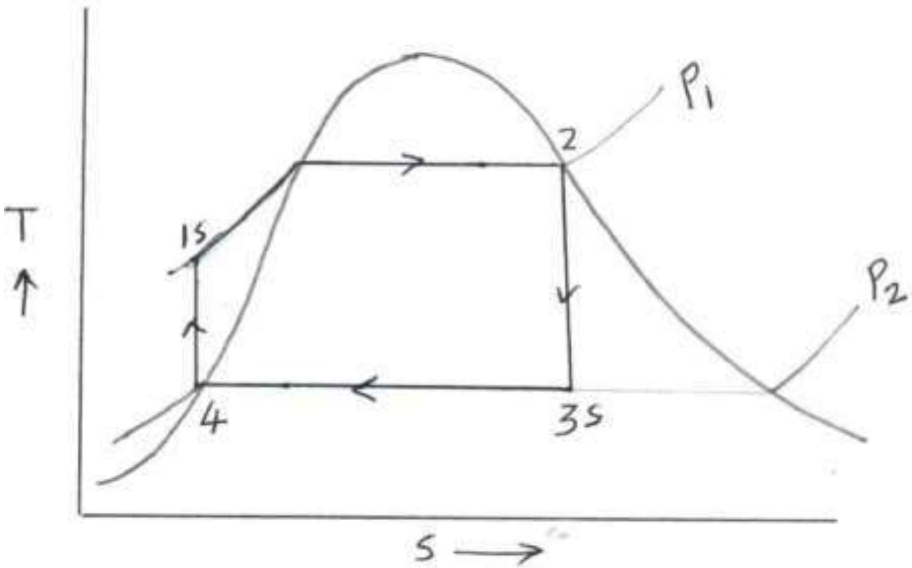
$W_P = 66.36$ kJ/kg

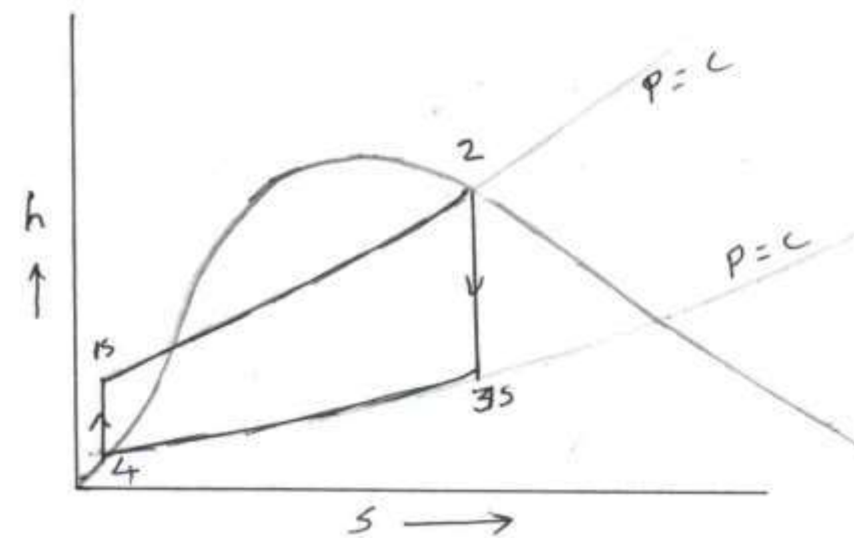
$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192\, x\, 10^{-3}\, kg/kJ$$

$$\text{work ratio} = \frac{Net.work.o/p.of.the.cycle}{Turbine.work}$$

$$r_w = \frac{\oint \delta w}{+ve\,work} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:





$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process,

work $W = -v\int dp$

$$\therefore W_P = h_{1S} - h_4 = v_{fP2} (P_1 - P_2)$$

$$= 0.001037 (17.5 - 0.75) \times 10^5 \times (1/1000)$$

$$= 1.737 \text{ kJ/kg}$$

$$\therefore h_{1S} = 1.737 + 384.25 = 385.99 \text{ kJ/kg}$$

Hence, turbine work = W_T = h_2 – h_3 =

522.47kJ/kg

Heat supplied = Q_H = h_2 – h_{1S}

= 2.794.1 – 385.99 =

2408.11 kJ/kg

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204 \ x \, 10^{-3} \, kg / kJ$$

Work ratio $= \frac{Net.work.o / p.of.the.cycle}{Turbine.work}$

$$r_w = \frac{522.47 - 1737}{522.47} = 0.9967$$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 kJ/kg$$

$$W_T = \eta_T W_T = 0.85\,(522.47) = 444.09 \text{ kJ/kg}$$

$$\therefore W_{net} = W_T - W_P = 442.06 \text{ kJ/kg}$$

$\therefore$% reduction in work output =

$$\frac{520.73 - 442.06}{520.73} = 15.11\%$$

$W_P = h_{1S} - h_4 \quad \therefore h_{1S} = 2.0435 + 384.25 = 386.29$ kJ/kg

$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81$ kJ/kg

$$\therefore \eta_{cycle} = \frac{442.06}{2407.81} = 0.1836$$

$\therefore$% reduction in cycle efficiency

$$= \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

Since steam is dry saturated at state 2, locate this state at the pressure $P_2 = 17.5$ bar on the saturation line and read the enthalpy at this state. This will give the value of h_2

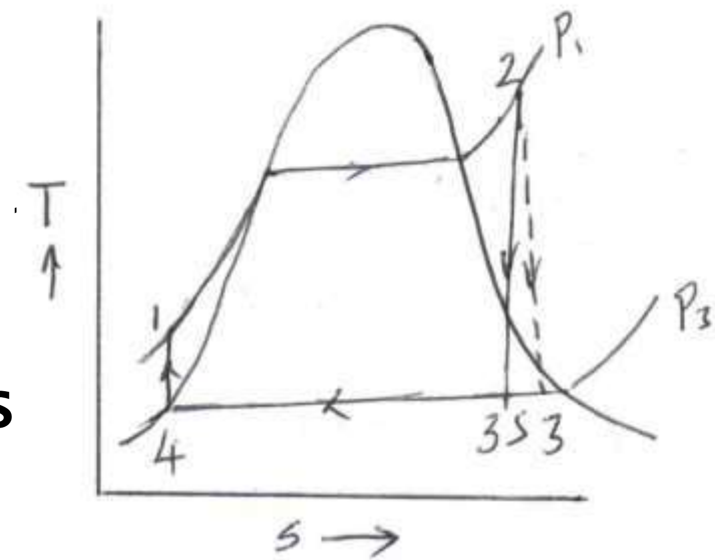
As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, P = 0.75 bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3.

The value of h_4 can be found from the steam tables at pressure, P = 0.75 bar, as $h_4 = h_{f4}$.

After finding the values of h_2, h_3 and h_4, apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

3. Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300^{0}C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η, neglecting pump work.

Solution:



P_1 = 10 bart_2 = 300^{0}C

P_3 = 0.1 bar , x_3 = 0.9,η_t =

η_{cycle} = ? Neglect W_P

From **superheated steam tables**

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For P_2 = 10 bar and t_2 = 300^{0}C,

h_2 = 3052.1 kJ/kg, s_2 = 7.1251 kJ/kg

From table A – 1, For $P_3 = 0.1$ bar

$t_S = 45.83^0C$ $h_f = 191.8$ $h_{fg} = 2392.9$

$S_f = 0.6493$ $S_{fg} = 7.5018$

Since $x_3 = 0.9$, $h_3 = h_{f4} + x_3\, h_{fg3}$

$= 191.8 + 0.9\,(2392.9)$

$= 2345.4$ kJ/kg

Also, since process 2-3s is (**wet steam** and) isentropic, $S_2 = S_{3S}$

i.e., $7.1251 = S_{fg4} + x_{3S}\, S_{fg3}$

$= 0.6493 + x_{3S}\,(7.5018)$

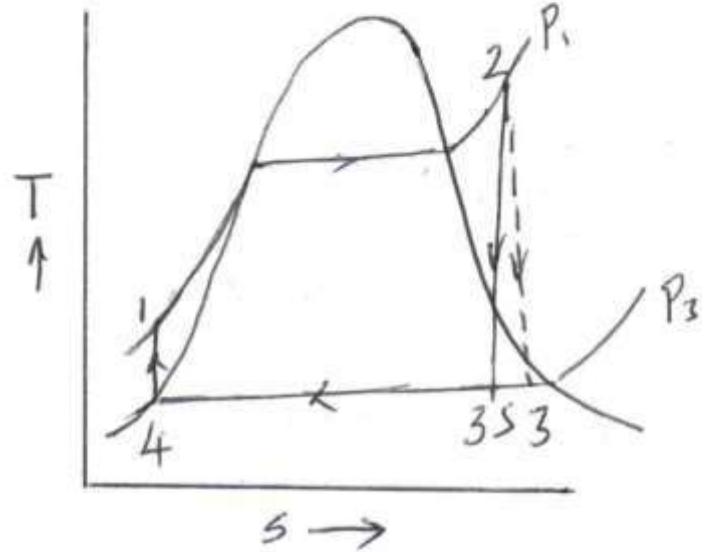
$\therefore x_{3S} = 0.863$

$\therefore h_{3S} = 191.8 + 0.863\,(2392.9) = 2257.43$ kJ/kg

$$\therefore \textit{Turbine efficiency}, \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

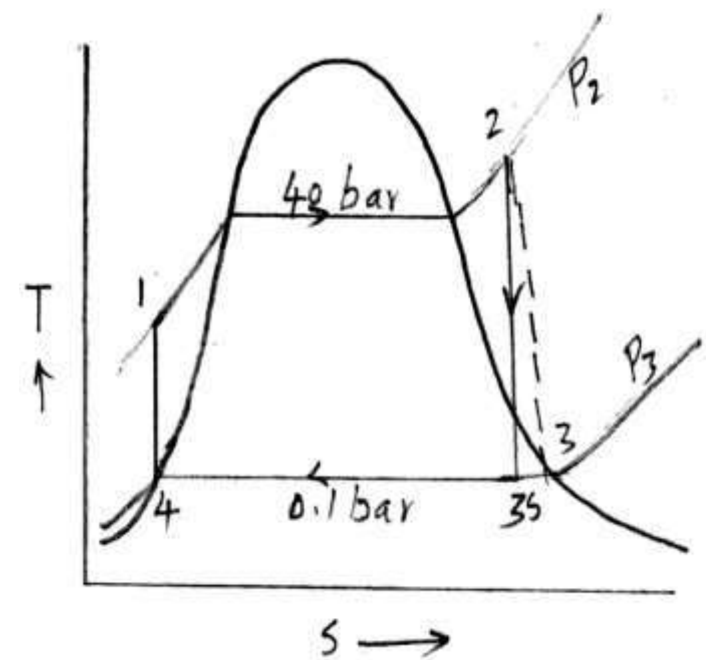
$$\eta_{cycle} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad but \quad h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \quad i.e., \quad 25\%$$



4. A 40 MW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400^0C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.

Solution:



$P_2 = 4$ MPa $= 40$ bar

$P_3 = 10$ KPa $= 0.1$ bar

$P = 40000$kW

$t_2 = 400^0$C $, \eta_t = 0.85$,

$\eta_{cycle} = ?$ & $x_3 = ?$

$$h_2 = h\Big|_{40bar,\,400^0C} = 3215.7 kJ/kg$$ and s_2 = 6.7733 kJ/kg-K

$$h_4 = h_f\Big|_{0.1bar} = 191.8 kJ/kg$$

Process 2-3s is isentropic i.e., $S_2 = S_{3S}$

$6.7733 = 0.6493 + x_{3S}\,(7.5018)$

$\therefore x_{3S} = 0.816$

$\therefore h_{3S} = h_{f3} + x_{3S}\,h_{fg3} = 191.8 + 0.816\,(2392.9)$

$= 2145.2$ kJ/kg

$$But \quad \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} \quad i.e., \quad 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$\therefore h_3 = 2305.8$ kJ/kg

$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9$ kJ/kg

$W_P = v\int dP$ $= 0.0010102\ (40 - 0.1)\ 10^5/10^2$

$= 4.031$ kJ/kg

$= h_1 - h_4$ $\therefore h_1 = 195.8$ kJ/kg

$$(i)\ \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{909.9 - 4.031}{(3215.7 - 195.8)} = 29.9\%$$

(ii) x_3 = ?

we have $2305.8 = 191.8 + x_3\ (2392.9)$

$\therefore x_3 = 0.88$

(iii) $P = \dot{m}\, W_{net}$

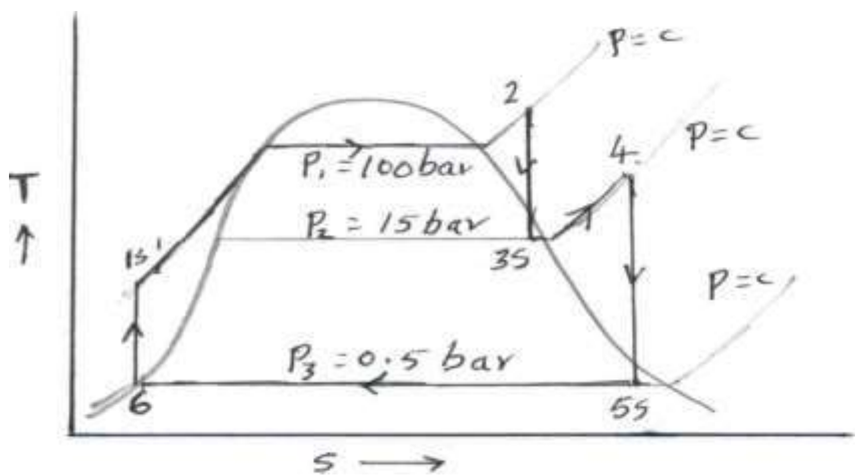
i.e., $40000 = \dot{m}\ (905.87)$

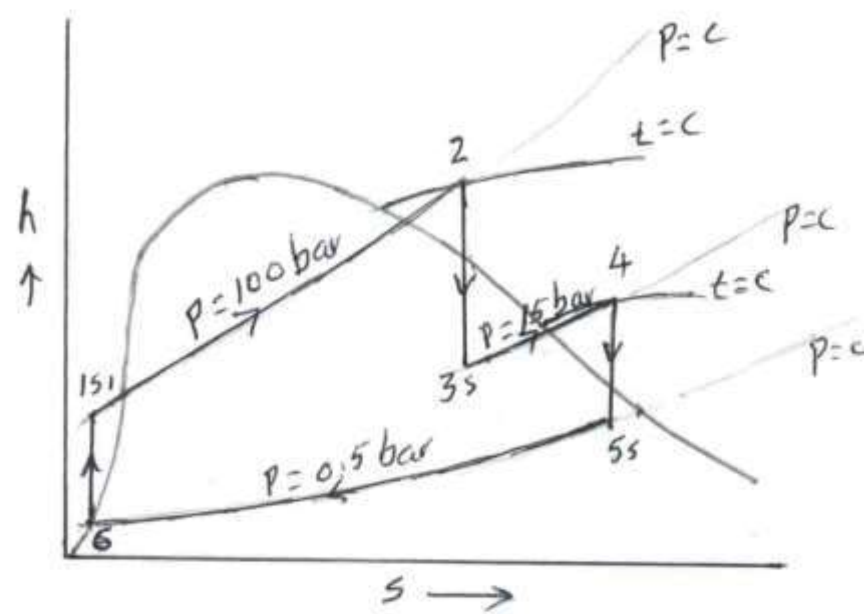
$\therefore \dot{m} = 44.2$ kg/s

$= 159120$ kg/hr

5. An ideal reheat cycle utilizes steam as the working fluid. Steam at 100 bar, 400^{0}C is expanded in the HP turbine to 15 bar. After this, it is reheated to 350^{0}C at 15 bar and is then expanded in the LP turbine to the condenser pressure of 0.5 bar. Determine the thermal η and steam rate.

Solution:





From steam tables

$P = 100$ bar $t = 400^0C$ = $v = 0.026408 m^3/kg$

$h = 3099.9$ kJ/kg

$S = 6.2182$ kJ/kg-K

$P = 15$ bar $t_S = 192.28^0C$, $v_f = 0.0011538 m^3/kg$,

$v_g = 0.13167 m^3/kg$

$h_f = 844.6$ kJ/kg,

$h_{fg} = 1845.3$ kJ/kg,

$h_g = 2789.9$ kJ/kg

$s_f = 2.3144$ kJ/kg-K,

$s_{fg} = 4.1262$ kJ/kg-K,

$s_g = 6.4406$ kJ/kg-K

At $P = 0.5$ bar & $t_S = 81.35^0C$,

$v_f = 0.0010301\ m^3/kg$, $v_g = 3.2401\ m^3/kg$

$h_f = 340.6$ kJ/kg, $h_{fg} = 2305.4$ kJ/kg, $h_g = 2646.0$ kJ/kg

$s_f = 1.0912$ kJ/kg-K, $s_{fg} = 6.5035$ kJ/kg-K, $s_g = 7.5947$ kJ/kg-K

$h_2 = 3099.9$ kJ/kg,

Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$

$6.2182 = 2.3144 + x_{3S}\ (4.1262) \quad \therefore x_{3S} = 0.946$

$\therefore h_{3S} = 844.6 + x_{3S}\ (1845.3) = 2590.44$ kJ/kg

$\therefore$Expansion of steam in the HP turbine $= h_2 - h_{3S}$

$= 3099.9 - 2590.44$

$= 509.46$ kJ/kg

$P = 15$ bar, $t = 350^0C$ = $v = 0.18653$
$h = 3148.7$
$s = 7.1044$

Expansion of steam in the LP cylinder = $h_4 – h_{5s}$

$$h_4 = 3148.7 \text{ kJ/kg}$$

To find h_{5s}:

We have $S_4 = S_{5s}$

$$7.1044 = S_{f5} + x_{5S} S_{fg5}$$

$$= 1.0912 + x_{5S} (6.5035)$$

$$\therefore x_{5S} = 0.925$$

$\therefore h_{5s} = 340.6 + 0.925 (2305.4) = 2473.09$ kJ/kg

$\therefore$ Expansion of steam in the LP turbine $= 3148.7 – 2473.09$

$= 675.61$ kJ/kg

$h_6 = h_f$ for $P_3 = 0.5$ bar i.e., $h_6 = 340.6$ kJ/kg

Pump work, $W_P = h_{1s} - h_6$

$= v_{f5}(P_3 - P_1) = 0.0010301\,(100 - 0.501 \times 10^5)$

$= 10.249$ kJ/kg

$\therefore h_{1s} = 350.85$ kJ/kg

$\therefore$ Heat supplied,

$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$

$= (3099.9 - 350.85) + (3148.7 - 2590.44)$

$= 2749.05$ kJ/kg $+ 558.26$

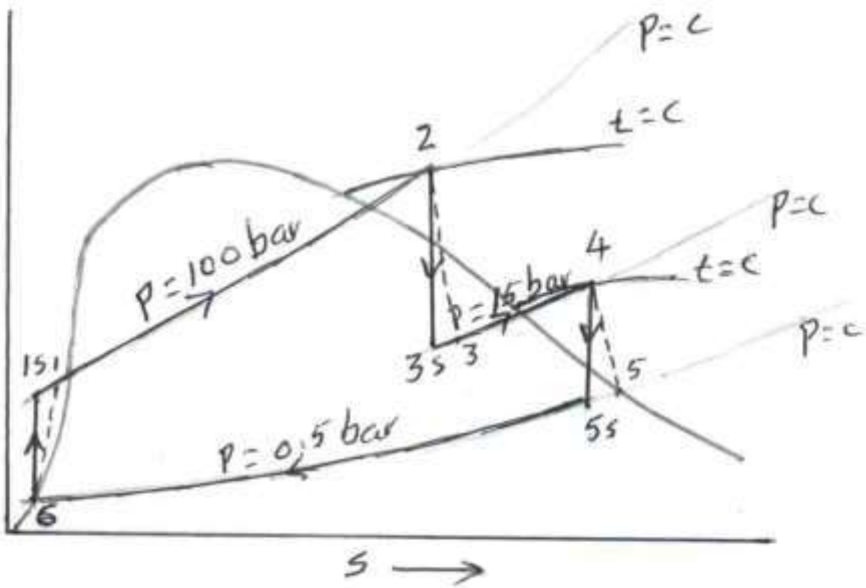
$= 3307.31$ kJ/kg

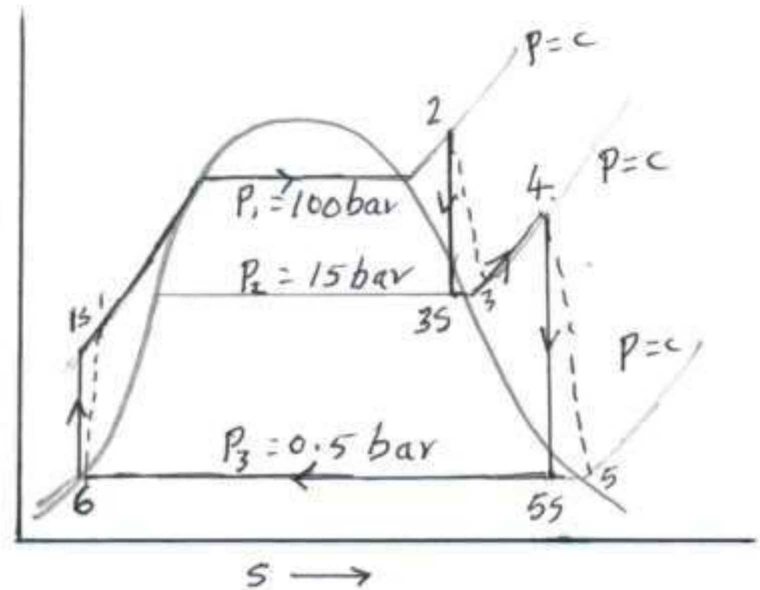
$$\therefore \quad \eta_{th} = \frac{W_{net}}{Q_H} = \frac{(W)_{HP} + (W)_{LP} - W_P}{Q_H}$$

$$= \frac{509.46 + 675.61 - 10.25}{3307.31} = 0.355$$

Steam rate, SSC $\displaystyle = \frac{3600}{W_{net}} = 3.064\,kg/kWh$

b) When η of the HP turbine, LP turbine and feed pump are 80%, 85% and 90% respectively.





$$\eta_{tHP} = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.8 = \frac{3099.9 - h_3}{3099.9 - 2590.44}$$

$$\therefore h_3 = 2692.33 \text{ kJ/kg}$$

$$\eta_{tLP} = \frac{h_4 - h_5}{h_4 - h_{5s}} = 0.85 = \frac{3148.7 - h_5}{3148.7 - 2473.09} \qquad \therefore h_5 = 2574.43 \text{ kJ/kg}$$

$$\eta_P = \frac{h_{15} - h_6}{h_1 - h_6} = 0.9 = \frac{350.85 - 340.6}{h_1 - 340.6} \qquad \therefore h_1 = 351.99 \text{ kJ/kg}$$

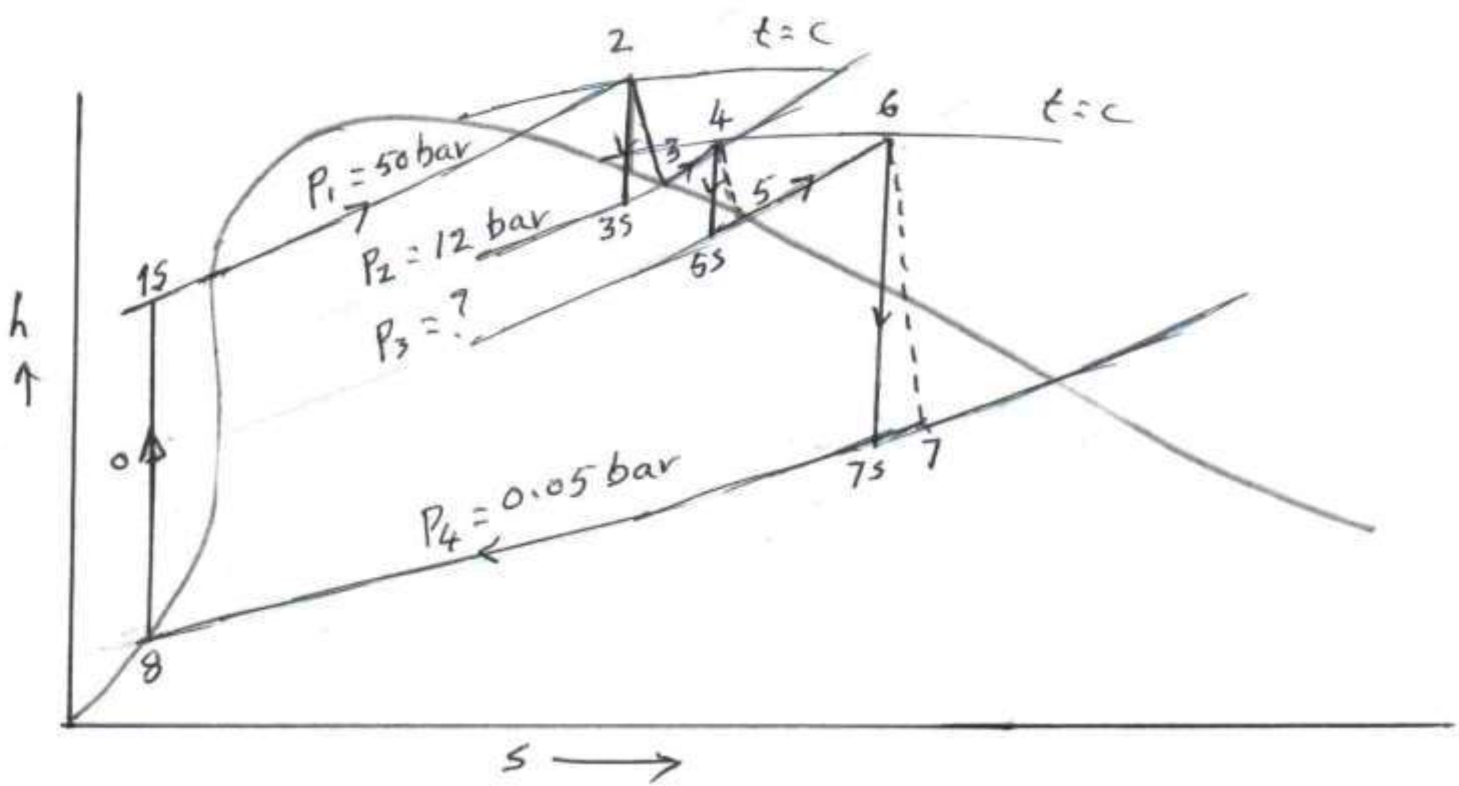
$$\therefore \eta_{th} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)}$$

$$= \frac{(3099.9 - 2692.33) + (3148.7 - 2574.43) - (351.99 - 340.6)}{(3099.9 - 351.99) + (3148.7 - 2692.33)}$$

= 0.303 or 30.3% $\qquad \therefore$SSC = 3.71 kg/kWh

Using Mollier-chart: h_2 = 3095 kJ/kg, h_{3s} = 2680 kJ/kg, h_4 = 3145 kJ/kg, h_5 = 2475 kJ/kg, h_6 = 340.6 kJ/kg (from steam tables), W_P = 10.249 kJ/kg stage, and is dry saturated at the stage exit. This is now reheated to 280^0C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^0C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



P_1 = 50 bar $t_2 = 350^0$C P_2 = 12 bar $t_4 = 280^0$C, $t_6 = 280^0$C P_3 = ? P_4 = 0.05 bar

From Mollier diagram

h_2 = 3070kJ/kg h_{3s} = 2755 kJ/kg h_3 = 2780 kJ/kg h_4 = 3008 kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$ = 0.921

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$$h_2 - h_3 = h_4 - h_5$$

$$\text{i.e., } 3070 - 2780 = 3008 - h_5$$

$$\therefore h_5 = 2718 \text{ kJ/kg}$$

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,

Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{kJ/kg}$$

$$\therefore \text{At a pressure 0.05 bar, } h_7 = h_{f7} + x_7\, h_{fg7}$$

$$2420.93 = 137.8 + x_7\,(2423.8)$$

$$\therefore x_7 = 0.941$$

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$

$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$

$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$W_P = 0.0010052\ (50 - 0.05)\ 10^2 = 5.021$ kJ/kg

$= h_8 - h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, $Q_H = (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)$

$= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718)$

$= 3475.18$ kJ/kg

$W_{net} = W_T – W_P = 1197.07 – 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m} \qquad \therefore T_m = 465.6 \text{ K}$$

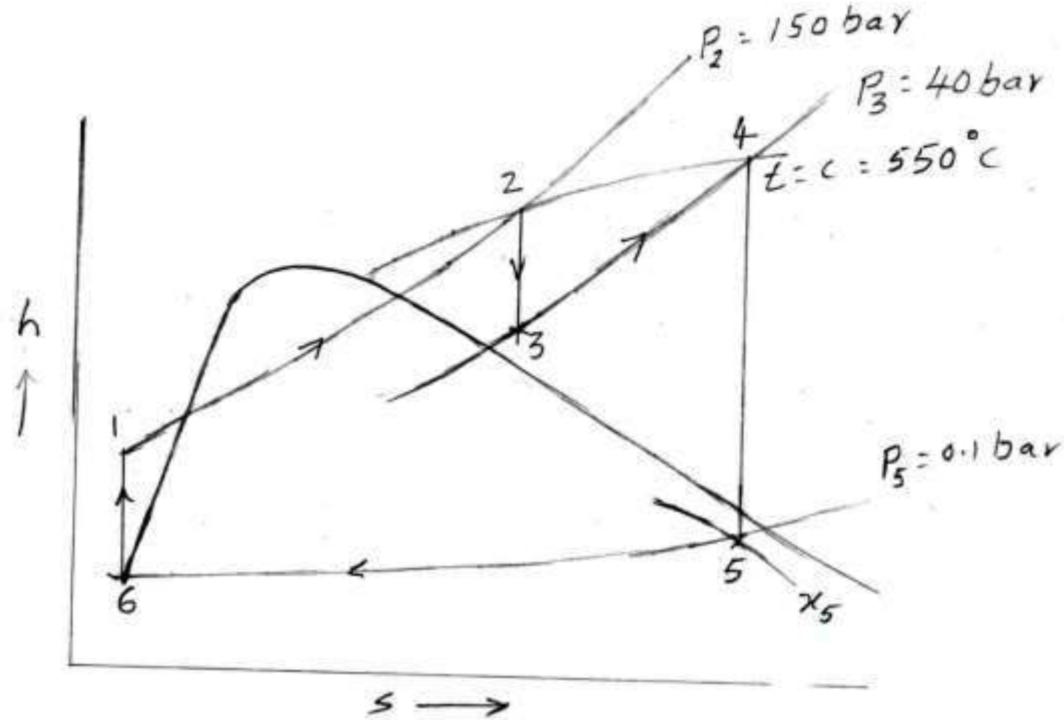
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02 kg / kWh$$

7. A steam power station uses the following cycle: Steam at boiler outlet – 150 bar; reheat at 40 bar, 550^{0}C; condenser at 0.1 bar. Using Mollier chart and assuming that all processes are ideal, find (i) quality at turbine exhaust (ii) cycle η (iii) steam rate.

Solution:



P_2 = 150 bar $t_2 = 550^0C$ P_3 = 40 bar $t_3 = 550^0C$

P_5 = 0.1 bar

From Mollier diagram i.e., h-s diagram

h_3 = 3050 kJ/kg

h_5 = 2290 kJ/kg

x_5 = 0.876 kJ/kg

h_6 can not determined from h-s diagram, hence steam tables are used.

$$h_2 = h\Big|_{150bar, 550^0 C} = 3450 kJ/kg$$

$$h_4 = h\Big|_{40bar, 550^0 C} = 3562 kJ/kg \qquad h_6 = h_f\Big|_{0.1bar} = 191.8 kJ/kg$$

Process 6-1 is isentropic pump work i.e., $W_{P1} = v\int dP$

$= 0.0010102\ (40 – 01)\ 10^5/10^3 = 4.031$ kJ/kg

$= (h_1 – h_6)$

$\therefore h_1 = 195.8$ kJ/kg

(i) Quality of steam at turbine exhaust = x_5 = 0.876

$$(ii)\ \eta_{cycle} = \frac{W_T - W_P}{Q_H}$$

Turbine work, W_T = $W_{T1} + W_{T2}$

= $(h_2 - h_3) + (h_4 - h_5)$

= (3450 – 3050) + (3562 – 2290)

= 1672 kJ/kg

$Q_H = Q_1 + Q_R$ = $(h_2 - h_1) + (h_4 - h_3)$

= (3450 – 195.8) + (3562 – 3050)

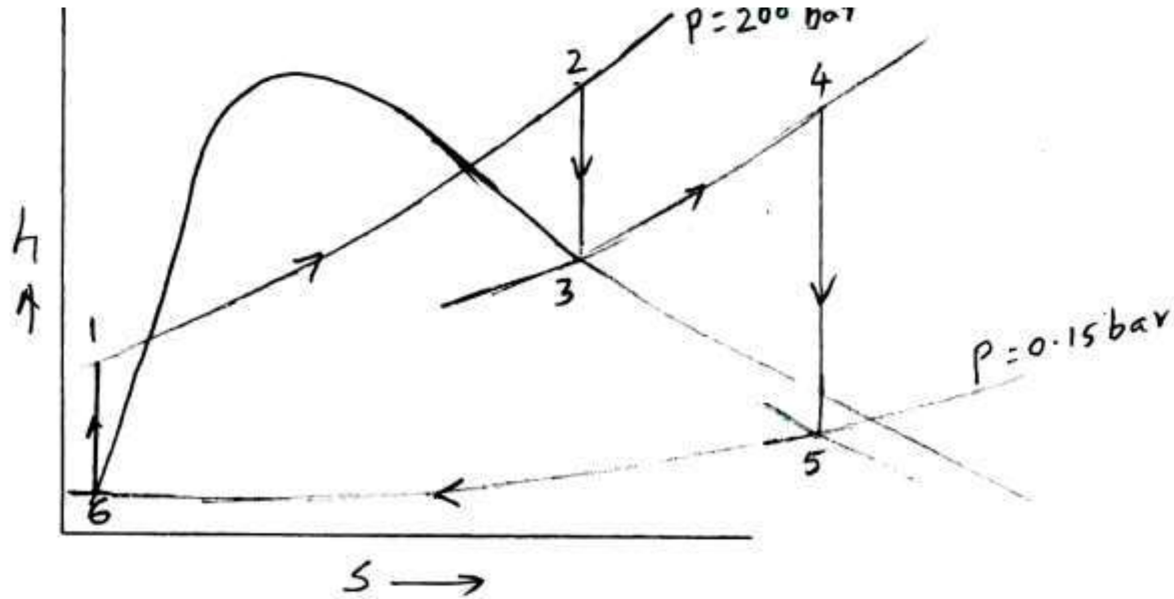
= 3766.2 kJ/kg

$$\therefore \eta_{cycle} = \frac{1672 - 4.031}{3766.2} = \frac{1667.97}{3766.2} = 0.443$$

$$(iii)\quad Steam\ rate = \frac{3600}{1667.97} = 2.16 kg/kWh$$

8. An ideal Rankine cycle with reheat is designed to operate according to the following specification. Pressure of steam at high pressure turbine = 20 MPa, Temperature of steam at high pressure turbine inlet = 550^{0}C, Temperature of steam at the end of reheat = 550^{0}C, Pressure of steam at the turbine exhaust = 15 KPa. Quality of steam at turbine exhaust = 90%. Determine (i) the pressure of steam in the reheater (ii) ratio of pump work to turbine work, (iii) ratio of heat rejection to heat addition, (iv) cycle η.

Solution:



$P_2 = 200$ bar $t_2 = 550^0C$ $t_4 = 550^0C$ $P_5 = 0.15$ bar $x_5 = 0.9$

From Mollier diagram,

$h_2 = 3370$ kJ/kg

$h_3 = 2800$ kJ/kg

$h_4 = 3580$ kJ/kg

$h_5 = 2410$ kJ/kg

$x_5 = 0.915$

$P_3 = P_4 = 28$ bar

But given in the data $x_5 = 0.9$

From steam tables $h_6 = 226$ kJ/kg

Pump work $W_P = v\int dP$

$= 0.001014\ (200 - 0.15)\ 10^5/10^3$

$= 20.26$ kJ/kg

But $W_P = h_1 - h_6$ $\therefore h_1 = 246.26$ kJ/kg

(i) Pressure of steam in the reheater = 28 bar

(ii) Turbine work $W_T = (h_2 - h_3) + (h_4 - h_5)$

$= (3370 - 2800) + (3580 - 2410)$

$= 1740$ kJ/kg

$$\therefore \text{Ratio of } \frac{W_P}{W_T} = 0.0116 \quad i.e., \quad 1.2\%$$

(iii) $Q_L = (h_5 - h_6) = (2410 - 226) = 2184$ kJ/kg

$Q_H = (h_2 - h_1) + (h_4 - h_3)$

$= (3370 - 226) + (3580 - 2800)$

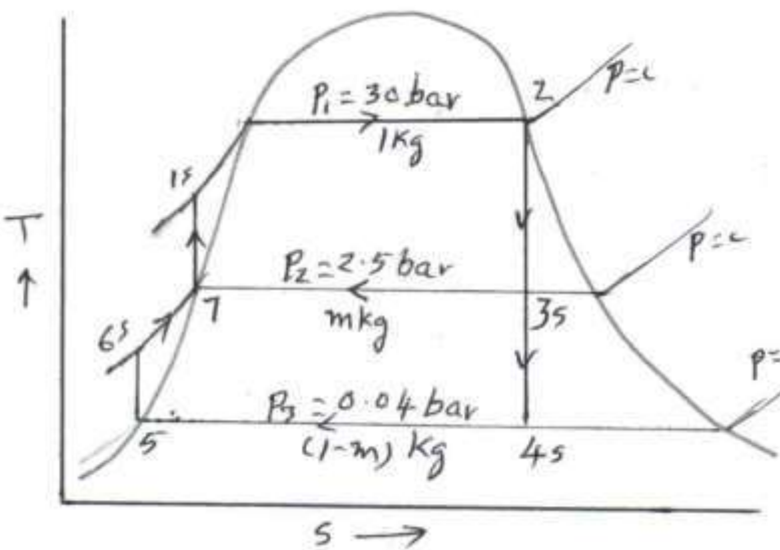
$= 3924$ kJ/kg

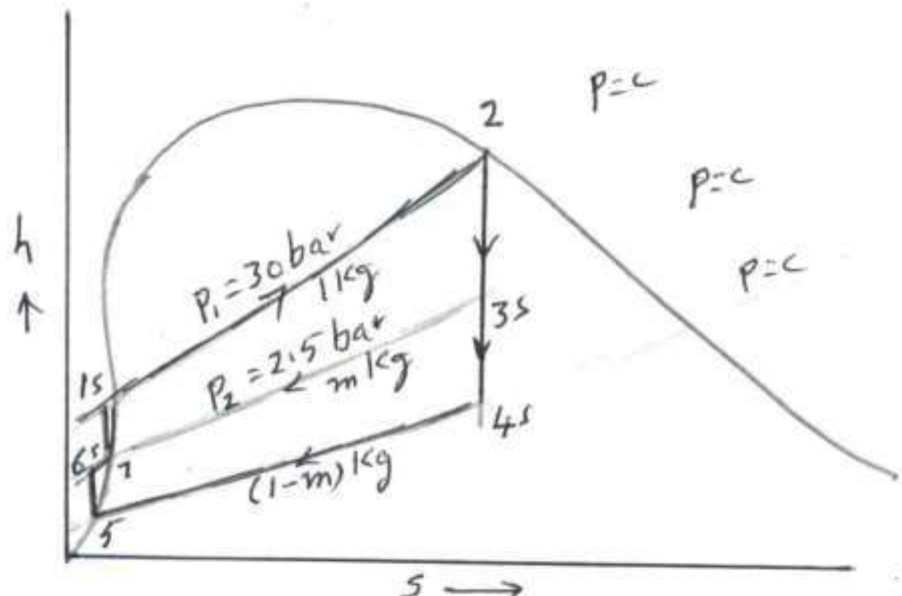
$$\therefore \frac{Q_L}{Q_H} = 0.5565 \quad i.e., \quad 55.65\%$$

$$(iv)\ \eta_{cycle} = \frac{W_{net}}{Q_{Total}} = \frac{(1740 - 20.26)}{3924} = 0.4383 \quad i.e., \quad 43.8\%$$

9. **An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.**

Solution:





$P_1 = 30$ bar $\quad P_2 = 2.5$ bar $\quad P_3 = 0.04$ bar

From steam tables, For $P_1 = 30$ bar, $h_2 = 2802.3$ kJ/kg,

$S_2 = 6.1838$ kJ/kg^{0}k

But $S_2 = S_{3s}$ i.e., $6.1838 = 1.6072 + x_3 (5.4448) \therefore x_3 = 0.841$

$\therefore h_3 = 535.4 + 0.841 (2281.0) \quad = 2452.68$ kJ/kg

Also $S_2 = S_{4s}$ $\quad$ i.e., $6.1838 = 0.4225 + x_4 (8.053) \quad \therefore x_4 = 0.715$

$\therefore h_4 = 121.4 + 0.715 (2433.1) \quad = 1862.1$ kJ/kg

At $P_3 = 0.04$ bar, $h_5 = 121.4$ kJ/kg, $\quad v_5 = 0.001004$ m^3/kg

$\therefore$ Condensate pump work $\quad = (h_6 – h_5) = v_5 (P_2 – P_3)$

$= 0.001004 (2.5 – 0.04) (10^5/10^3)$

$= 0.247$ kJ/kg

$\therefore h_6 = 0.247 + 121.4 = 121.65$ kJ/kg

Similarly, $h_1 = h_7 + v_7 (P_1 – P_2) (10^5/10^3)$
$= 535.4 + 0.0010676 (30 – 2.5) 10^2$
$= 538.34$ kJ/kg

a) Mass of the bled steam:

Applying the energy balance to the feed water heater

$$mh_3 + (1 – m) h_6 = 1 (h_7)$$

$$\therefore m = \frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)} = 0.177 kg/kg$$ of steam

b) Thermal η:

Turbine work, $W_T = 1 (h_2 – h_{3s}) + (1 – m) (h_3 – h_{4s})$
$(2802.3 – 2452.65) + (1 – 0.177) (2452.68 – 1862.1)$
$= 835.67$ kJ/kg

Pump work, $W_P = (1 - m)(h_{6s} - h_5) + 1(h_{1s} - h_7)$

$= (1 - 0.177)(121.65 - 121.4) + 1(538.34 - 535.4)$

$= 3.146$ kJ/kg

$\therefore W_{net} = W_T - W_P = 832.52$ kJ/kg

Heat supplied, $Q_H = 1(h_2 - h_{1s}) = 1(2802.3 - 538.34)$

$= 2263.96$ kJ/kg

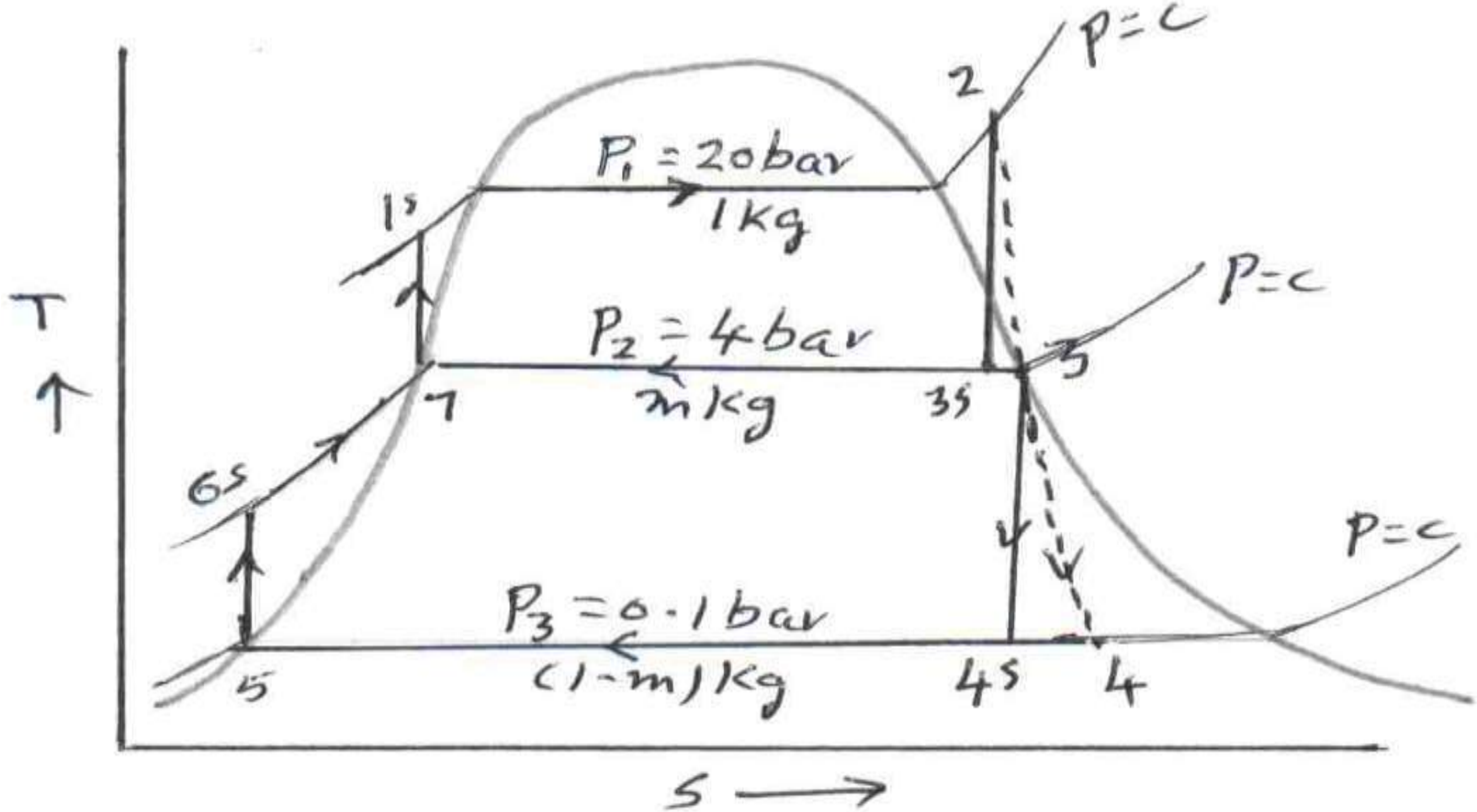
$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{832.52}{2263.96} = 0.368$$ or **36.8%**

c) **SSC:**

$$SSC = \frac{3600}{W_{net}} = 4.324 kg / kWh$$

10. Steam at 20 bar and 300 ^{0}C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find: a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η.

Solution:



P=C
2
P1 = 20bar
1kg
1s
P=C
T
P2 = 4bar
3
mkg
3s
7
6s
P=C
P3 = 0.1bar
(1-m)kg
5
4s
4
s

P_1 = 20 bar $t_1 = 300^0C$ P_2 = 4 bar P_3 = 0.1 bar

From steam tables,

For P_1 = 20 bar and $t_1 = 300^0C$

v_2 = 0.12550 h_2 = 3025.0 S_2 = 6.7696

For P_2 = 4 bar, h_3 = 2737.6, t_s = 143.63

h_f = 604.7, h_{fg} = 2132.9, S_f = 1.7764, S_{fg} = 5.1179, S_g = 6.8943

For P_2 = 0.1 bar, 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511

We have, $S_2 = S_{3s}$ i.e., 6.7696 = 1.7764 + x_3 (5.1179)

$\therefore x_3 = 0.976$

$\therefore h_{3s}$ = 604.7 + 0.976 (2132.9) = 2685.63 kJ/kg

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$S_3 = S_{4s}$ i.e., 6.8943 = 0.6493 + x_4 (7.5018)

$\therefore x_{4s}$ = 0.832

$\therefore h_{4s}$ = 191.8 + 0.832 (2392.9) = 2183.81kJ/kg

But η_t is same before and after bleeding i.e.,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

i.e., $0.847 = \frac{2737.6 - h_4}{2737.6 - 2183.81}$

$\therefore h_4 = 2268.54$ kJ/kg

$\therefore h_4 = h_{f4} + x_4 h_{fg4} \therefore x_4 = 0.868$

b) Applying energy balance to open heater,

$mh_3 + (1 - m) h_{6s} = 1 (h_7)$

Condensate pump work, $(h_{6s} - h_5) = v_5 (P_3 - P_2)$

$= 0.0010102 (3.9) 10^2$

$= 0.394$ kJ/kg

$\therefore h_{6s} = 191.8 + 0.394 = 192.19$ kJ/kg

Similarly, $h_{1s} = h_7 + v_7 (P_1 - P_2)$

$= 604.7 + -.0010839 (16) 10^2 = 606.43$ kJ/kg

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6} \qquad \therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

c) Power output or $W_T = (h_2 – h_3) + (1 – m)(h_3 – h_4)$

$$(3025 – 2737.6) + (1 – 0.162)(2737.6 – 2268.54)$$

$$= 680.44 \text{ kJ/kg}$$

For 1kg/s of steam, $W_T = 680.44$ kW

d) Overall thermal efficiency, $\eta_0 = \frac{W_{net}}{Q_H}$

$W_P = (1 – m)(h_{6s} – h_5) + 1(h_{1s} – h_7)$

$= (1 – 0162)(192.19 – 191.8) + 1(606.43 – 604.7)$

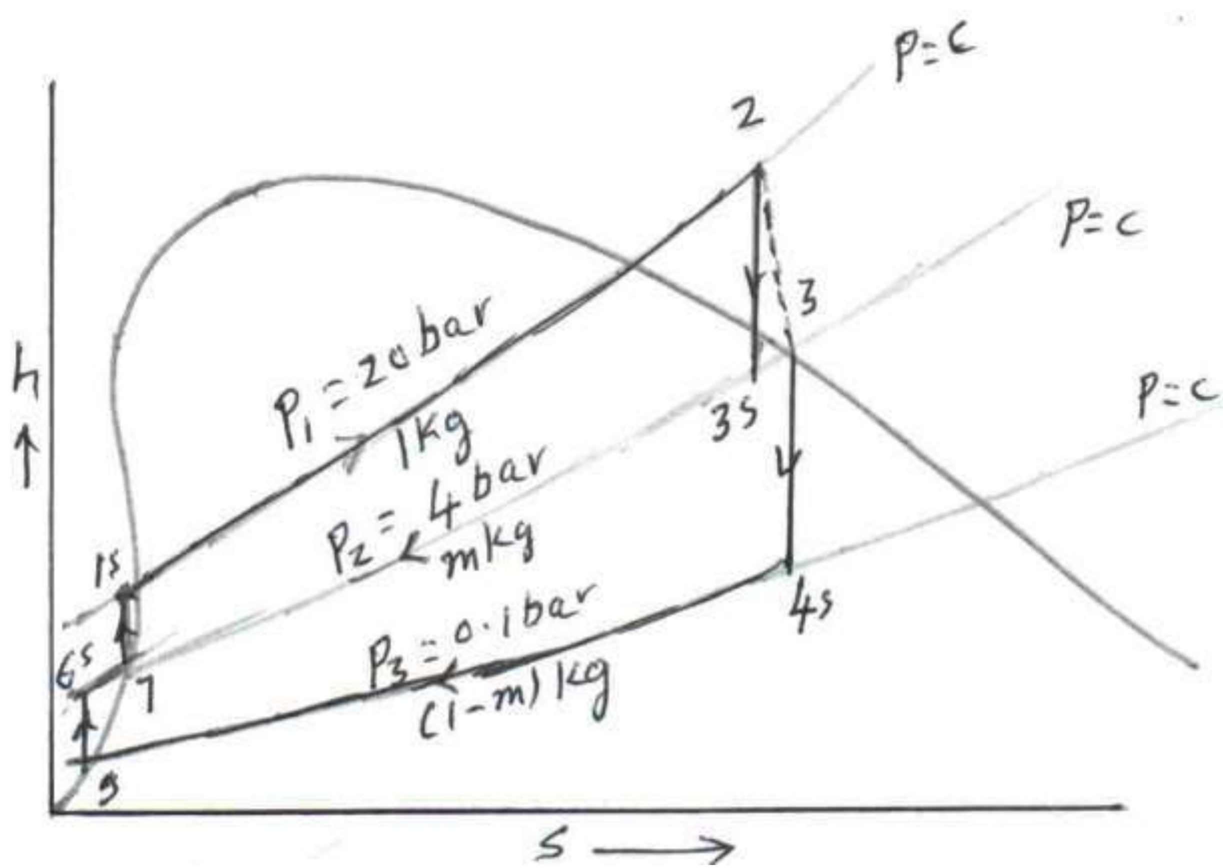
$= 2.057$ kJ/kg

$W_{net} = 680.44 – 2.057 = 678.38$ kJ/kg

$Q_H = 1(h_2 – h_{1s}) = (3025 – 606.43) = 2418.57$ kJ/kg

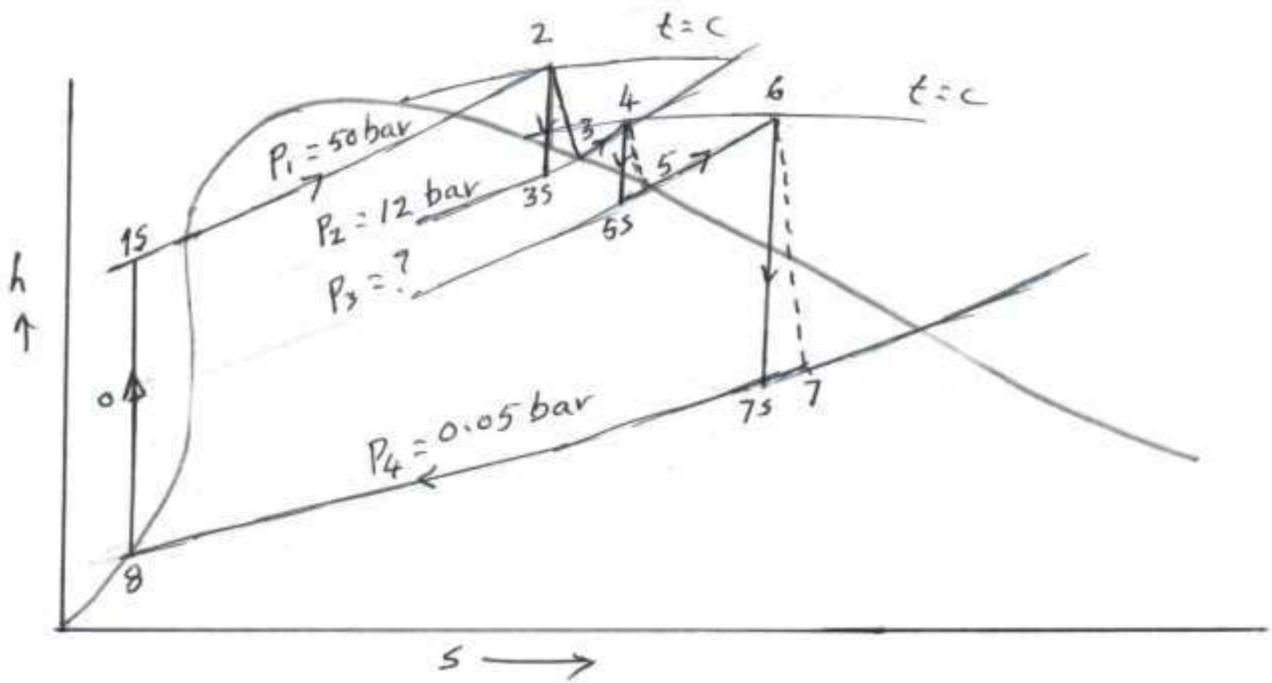
$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

Using Moiller Diagram



11. Steam at 50 bar, 350^0C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^0C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^0C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



P_1 = 50 bar $t_2 = 350^0$C P_2 = 12 bar $t_4 = 280^0$C,

$t_6 = 280^0$C P_3 = ? P_4 = 0.05 bar

From Mollier diagram

h_2 = 3070kJ/kg h_{3s} = 2755 kJ/kg h_3 = 2780 kJ/kg

h_4 = 3008 kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$ = 0.921

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$$h_2 - h_3 = h_4 - h_5$$

i.e., $3070 - 2780 = 3008 - h_5$ $\quad \therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,

Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7 h_{fg7}$

$2420.93 = 137.8 + x_7 (2423.8)$ $\quad \therefore x_7 = 0.941$

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$

$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$

$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

$\therefore$ Total power output /kg of steam = 1197.07 kW

For P_4 = 0.05 bar from steam tables, h_8 = 137.8 kJ/kg;

$W_P = 0.0010052\ (50 - 0.05)\ 10^2 = 5.021$ kJ/kg $= h_8 - h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, $Q_H = (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)$

$= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718) = 3475.18$ kJ/kg

$W_{net} = W_T - W_P = 1197.07 - 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$ $\therefore T_m = 465.6$ K

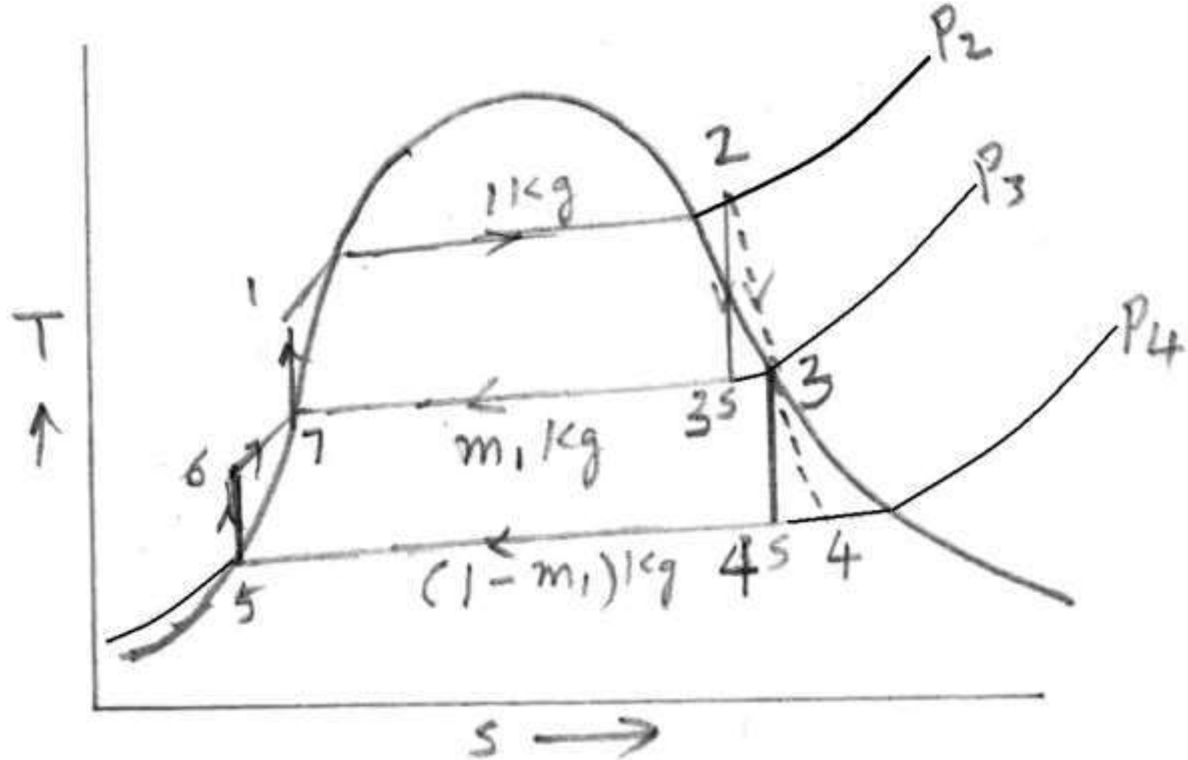
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02kg / kWh$$

12.Steam at 30 bar and 350^0C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η, (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η.

Solution:



P_2 = 30 bar t_2 = 350^{0}C P_3 = 4 bar P_4 = 0.1 bar

$h_3 = h_g$ at P_3 = 4 bar, = 2737.6 kJ/kg

From superheated steam tables,

$$h_2 = h_3 = h_g\Big|_{P_3=4bar} = 2737.6\ kJ/kg$$

$$h_2 = h\Big|_{P_2=30bar\ \&\ t_2=350^0 C} = 3117.5\ kJ/kg$$ and $S_2 = 6.7471$ kJ/kg-K

$$h_5 = h_f\Big|_{P_5=0.1bar} = 191.8\ kJ/kg$$

$$h_7 = h_f\Big|_{P_7=4bar} = 604.7\ kJ/kg$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$

$$6.7471 = 1.7764 + x_{3S}\ (5.1179)$$

$$\therefore x_{3S} = 0.971$$

$$\therefore h_{3S} = h_{f3} + x_{3S}\ h_{fg3}$$

$$= 604.7 + 0.971\ (2132.9)$$

$$= 2676.25\ kJ/kg$$

Process 3-4s is isentropic i.e., $S_3 = S_{4S}$

i.e., $6.8943 = 0.6493 + x_{4S}\,(7.5018)$

$\therefore x_{4S} = 0.832$

$\therefore h_{4S} = 191.8 + 0.832\,(2392.9) = 2183.8$ kJ/kg

Given, η_t (before bleeding) = η_t (after bleeding)

We have, η_t (before bleeding) $= \dfrac{h_2 - h_3}{h_2 - h_{3S}} = \dfrac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33 kJ/kg$$

But $h_4 = h_{f4} + x_4\, h_{fg4}$

$2261.33 = 191.8 + x_4\,(2392.9)$

$\therefore x_4 = 0.865$

i.e., Dryness fraction at entry to condenser = x_4 = **0.865**

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$mh_3 + (1 - m)\, h_6 = h_7$

We have $W_{P1} = (h_6 - h_5) = v\int dP$

$= 0.0010102\ (4 - 0.1)\ 10^5/10^3$

$= 0.394$ kJ/kg

$\therefore h_6 = 0.394 + 191.8 = 192.19$ kJ/kg

Substituting,

$m\,(2737.6) + (1 - m)\, 192.19 = 604.7$

$\therefore m = 0.162$ kg

Also, $W_{P2} = (h_1 - h_7) = v\int dP$

$= 0.0010839 \times (30 - 4)\ 10^2$

$= 2.82$ kJ/kg

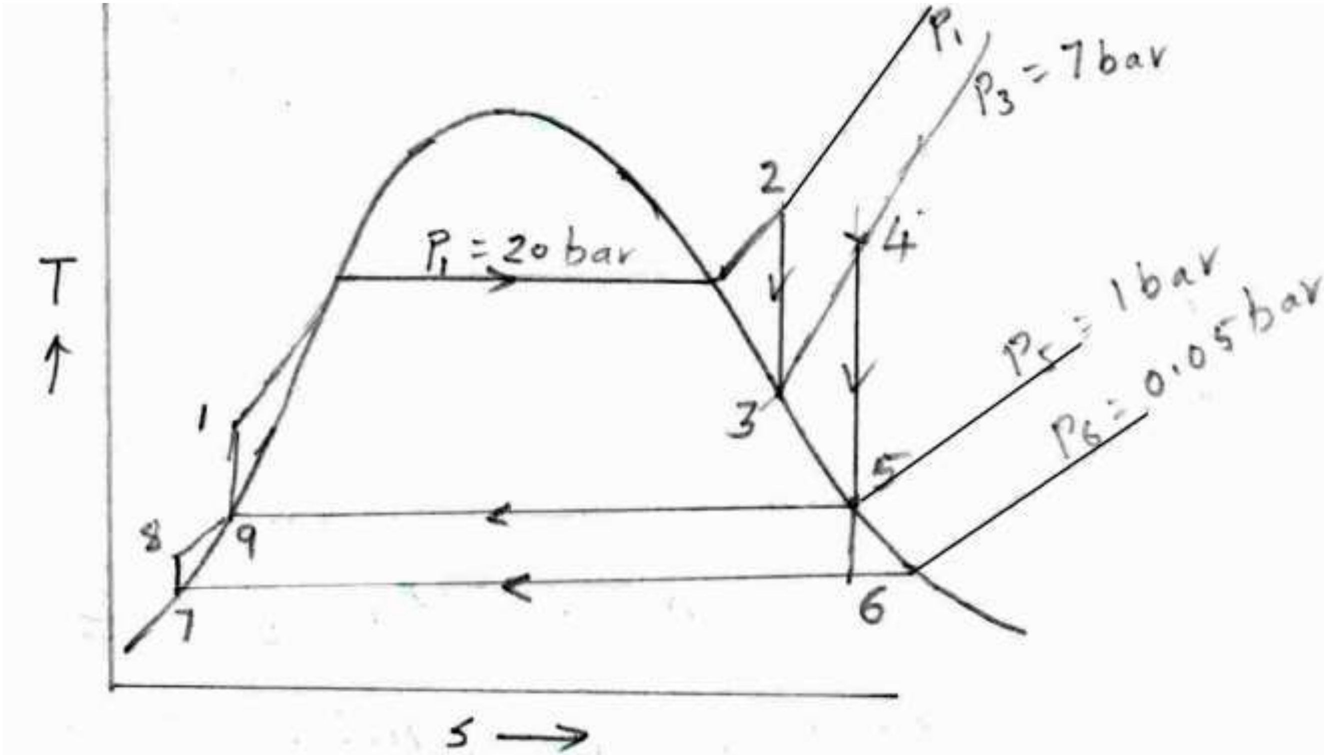
$\therefore h_1 = 2.82 + 604.7 = 607.52$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_T = W_P}{Q_H} = \frac{\left[(h_2 - h_3) + (1-m)(h_3 - h_4)\right] - \left[(1-m)(h_6 - h_5) + (h_1 - h_2)\right]}{(h_2 - h_1)}$$

η_{cycle} **= 0.31**

13. In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300^0C. After expansion to 7 bar, the steam is reheated to 300^0C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η, (ii) specific steam consumption, (iii) quality of steam entering condenser.

- Solution:



$$h_2 = h\Big|_{20bar, 300^0 C} = 3025 kJ/kg$$

and s_2 = 6.7696kJ/kg-K

Process 2-3 is isentropic i.e., $S_2 = S_3$

$6.7696 = 1.9918 + x_3 (4.7134)$

$\therefore x_3 = 1.014$

i.e., state 3 can be approximated as dry saturated.

$$\therefore h_3 = h\Big|_{7bar, dry sat.} = 2762 \text{kJ/kg}$$

$$\therefore h_4 = h\Big|_{7bar, 300^0 C} = 3059.8 kJ/kg$$ and s_4 = 7.2997 kJ/kg-K

Process 4-5 is isentropic, $S_4 = S_5$

i.e., 7.6798 = 0.4763 + x_5 (7.9197)

$\therefore x_5 = 0.909$

$\therefore h_5$ = 137.8 + 0.909 (2423.8) = 2342.41 kJ/kg

$$h_6 = h_f\Big|_{0.05bar} = 137.8 kJ/kg$$ $h_7 = h_6$ (since W_{P1} is neglected)

$$h_8 = h_f\Big|_{6.4bar} = 681.1 kJ/kg$$

$h_1 = h_8$ (since W_{P2} is neglected)

(ii) Applying energy balance to FWH,

$$mh_3 + (1 - m)\, h_7 = h_8$$

$$m\,(2758.1) + (1 - m)\, 137.8 = 681.1$$

$$\therefore m = 0.313 \text{ kg/kg of steam}$$

(iii) $W_1 = W_{HP} = (h_2 - h_3) = (3398.8 - 2758.1 = 640.7$ kJ/kg

$$W_2 = W_{LP} = (1 - m)\,(h_4 - h_5)$$

$$= (1 - 0.313)\,(3269.96 - 2342.41) = 637.2 \text{ kJ/kg}$$

$$\therefore W_{net} = W_1 + W_2 = 1277.9 \text{ kJ/kg}$$

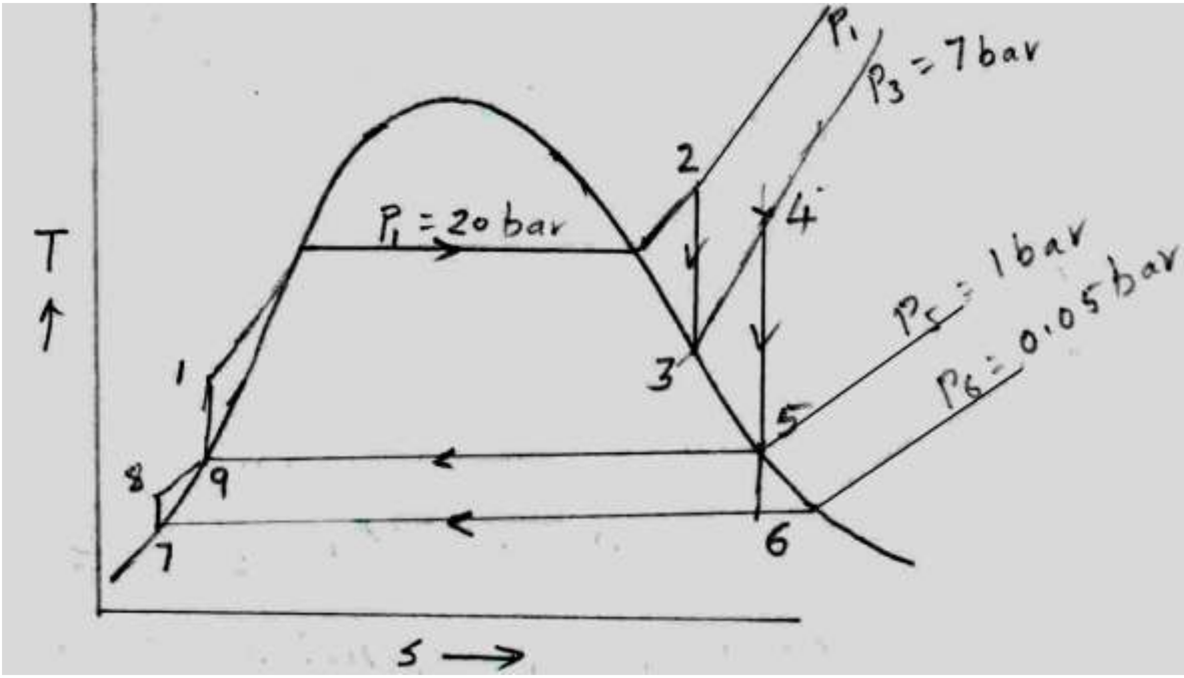
$$\therefore \text{Steam flow rate through HP turbine} = \frac{Power}{W_{net}} = \frac{80000}{1277.9} = 62.6\,kg/s$$

(iv) η_{cycle} = ? $Q_H = (h_2 - h_1) + (1 - m)\,(h_4 - h_3) = 3069.35$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

15. In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400^0C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may neglected.

- Solution:



$h_2 = h\Big|_{20bar, 300^0 C} = 3025 kJ/kg$ and $s_2 = 6.7696$ kJ/kg-K

Process 2-3 is isentropic

i.e., $S_2 = S_3$

$6.7696 = 1.9918 + x_3 (4.7134)$

$\therefore x_3 = 1.014$

i.e., state 3 can be approximated as dry saturated.

$$\therefore h_3 = h\Big|_{7\,bar,\,dry\,sat.} = 2762 kJ/kg$$

$$\therefore h_4 = h\Big|_{7\,bar,\,300^0 C} = 3059.8 kJ/kg$$ and s_4 = 7.2997 kJ/kg-K

Process 4–5 is isentropic i.e., $S_4 = S_5$

$7.2997 = 1.3027 + x_5 (6.0571)$ $\therefore x_5 = 0.99$

$\therefore h_5 = h_{f5} + x_5 h_{fg5} = 417.5 + 0.99 (2257.9) = 2652.9$ kJ/kg

Process 5–6 is isentropic i.e., $S_5 = S_6$

$7.2997 = 0.4763 + x_6 (7.9197)$ $\therefore x_6 = 0.862$

$\therefore h_6 = 137.8 + 0.862 (2423.8) = 2226.1$ kJ/kg

$h_7 = = 137.8$ kJ/kg

Neglecting W_{P1},

$h_8 = h_7$, Also neglecting W_{P2}, $h_9 = h_1 = 417.5$ kJ/kg

Applying energy balance to FWH

$mh_5 + (1 - m) h_8 = h_9$

i.e., $m (2652.9) + (1 - m) 137.8 = 417.5$

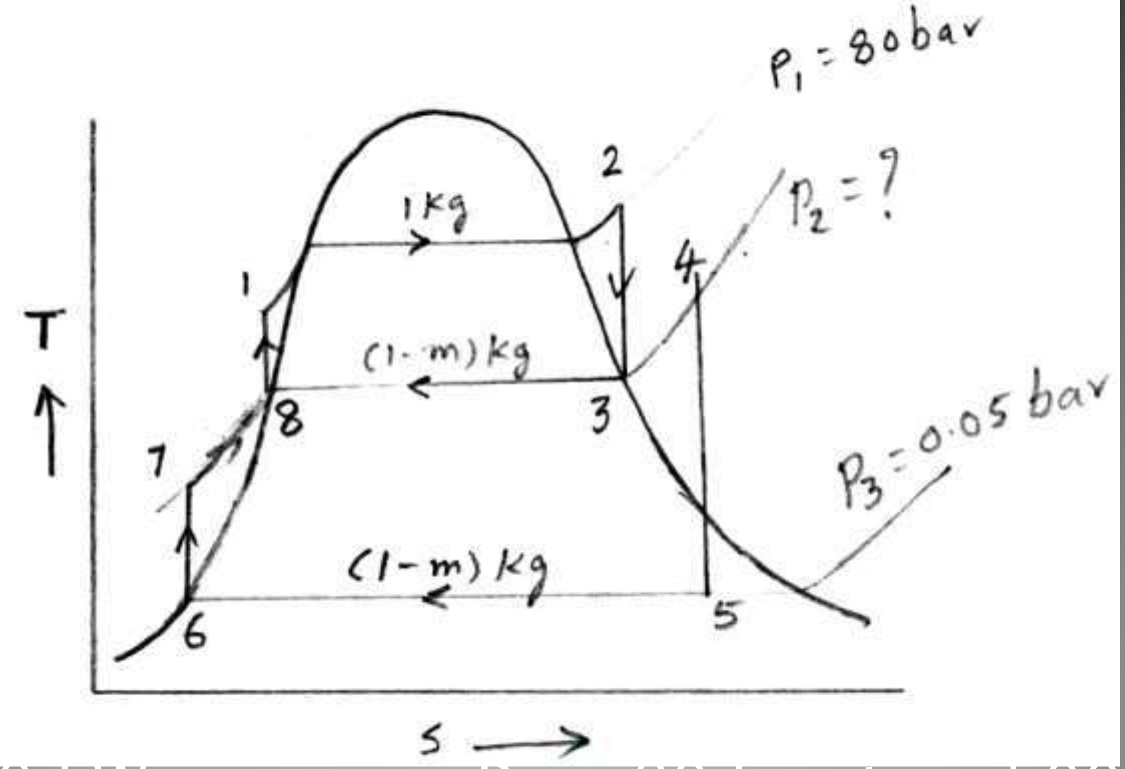
$\therefore m = 0.111$ kg/kg of steam

$$(i) \quad \eta_C = \frac{(h_2 - h_3) + (h_4 - h_5) + (1 - m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$$

$$(ii) \quad SSC = \frac{3600}{W_{net}} = 3.57 kg/kWh$$

(iii) Quality of steam entering condenser, $x_6 = 0.862$

16. The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500^0C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400^0C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2, (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η. Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.



P = 80000 kW P$_1$ = 80 bar t$_2$ = 500^{0}C P$_2$ = ? t$_3$ = 400^{0}C P$_3$ = 0.05 bar m = ? η_{cycle} = ?

$$h_2 = h\Big|_{80bar,500^0C} = 3398.8 kJ/kg$$

and s$_2$ = 6.7262

Process 2–3 is isentropic i.e., S$_2$ = S$_3$ = 6.7262 kJ/kg-K

Given state 3 is dry saturated i.e., S$_3$ = 6.7262 = $S_g\Big|_{P_2}$

From table A – 1, for dry saturated steam, at P = 6.0 bar, S_g = 6.7575 and at P = 7.0 bar, S_g = 6.7052

Using linear interpolation,

$$\Delta P = \frac{6.0 - 7.0}{(6.7575 - 6.7052)} x(6.7262 - 6.7052) = 0.402\ bar$$

$\therefore$(i) P_2 = 6 + 0.402 = 6.402 bar

$$\therefore h_3 = h\Big|_{P_2 = 6.4 bar}$$

From table A – 1, For P = 6 bar, h_g = 2755.5, S_g = 6.7575

For P = 7 bar, h_g = 2762.0 S_g = 6.7052

$$\therefore For\ P = 6.4\ bar \Rightarrow \frac{2762 - 2755.5}{1} x(0.4) + 2755.5 = 2758.1 kJ/kg$$

$\therefore h_3$ = 2758.1 kJ/kg